Causal reasoning and inference with causal Bayes nets

Alexander Gebharter

Duesseldorf Center for Logic and Philosophy of Science
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28.04.2016

Introduction

- The theory of causal Bayes nets (CBNs) can be seen as a non-reductionist probabilistic theory of causation.
- In classical (reductionist) theories of causation, causation is explicitly defined.
- Causation is *not* defined within the theory of CBNs.
- Causation is only implicitly characterized (by several axioms).
- Causal structures are assumed to produce probabilistic footprints by whose means they can (in principle) be identified.
- The theory provides the best explanation for certain empirical phenomena and the whole theory is empirically testable.

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- Introduction
- Causal Bayes nets
- Intervention and observation
- Causal reasoning with causal Bayes nets
- Causal discovery with causal Bayes nets

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Causal Bayes nets

Definition (probabilistic dependence/independence)

$Dep(X, Y|Z)$ iff $P(y|x, z) \neq P(y|z)$ for **some** X-, Y-, and Z-values x, y, and z, respectively, and $P(x, z) > 0$.

$Indep(X, Y|Z)$ iff $P(y|x, z) = P(y|z)$ for **all** X-, Y-, and Z-values x, y, and z, respectively, or $P(x, z) = 0$.

$(In)Dep(X, Y)$ iff $(In)Dep(X, Y|\emptyset)$

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Causal Bayes nets

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- $G = \langle V, E \rangle$ is a directed acyclic graph (DAG).

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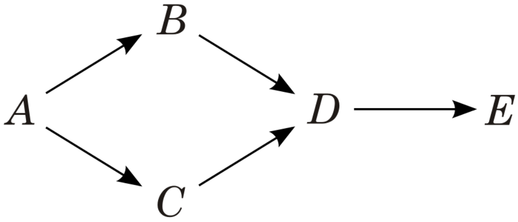
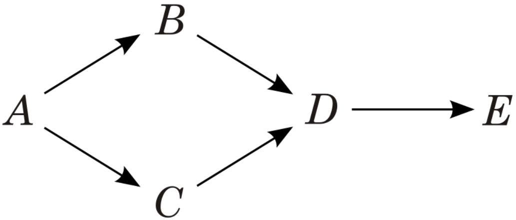


Causal Bayes nets



- π is a causal path between X and Y
- X is a direct cause/causal parent of Y
- X is a (direct or indirect) cause of Y
- X is an intermediate cause on π
- Z is a common cause of X and Y
- Z is a common effect (collider) of X and Y

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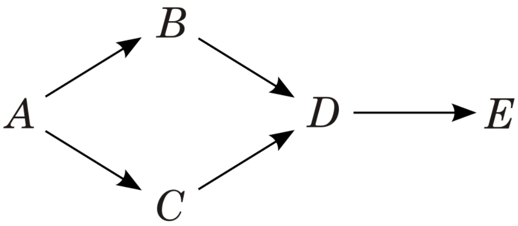

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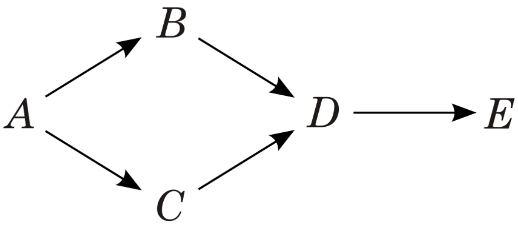

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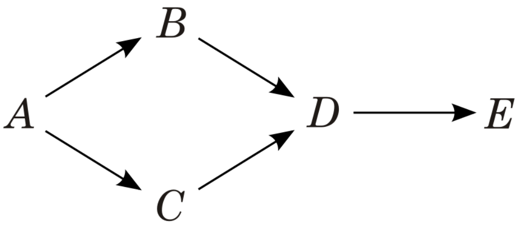

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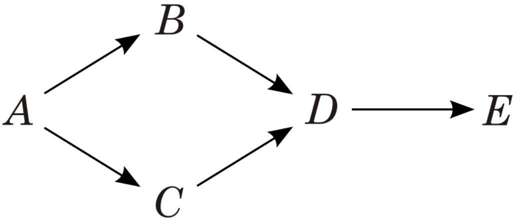

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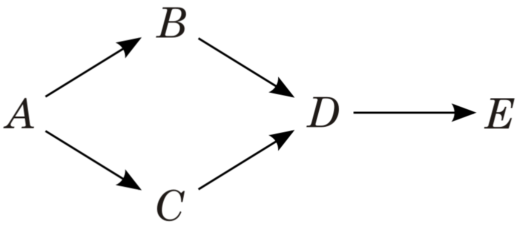

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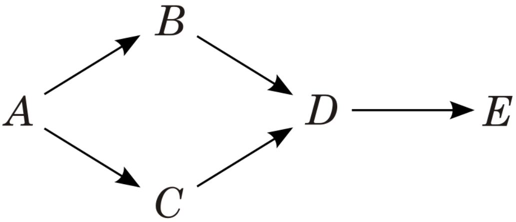


Definition (d-connection/d-separation)

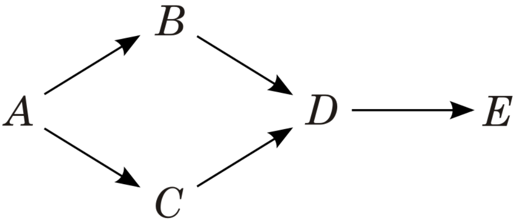
X and Y are d-connected by $Z \subseteq V \backslash \{X, Y\}$ if and only if X and Y are connected by a causal path π such that

(i) no non-collider on π is in Z, and

(ii) every collider on π is in Z or has an effect in Z.

X and Y are d-separated by Z iff they are not d-connected by Z.

Causal Bayes nets



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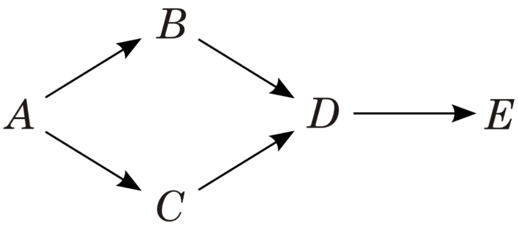

Definition (*d*-connection/*d*-separation)

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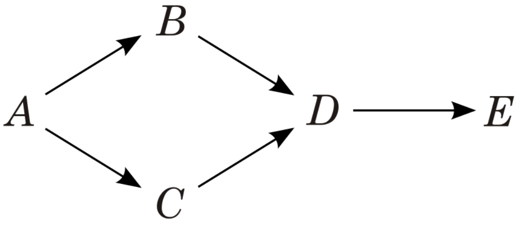

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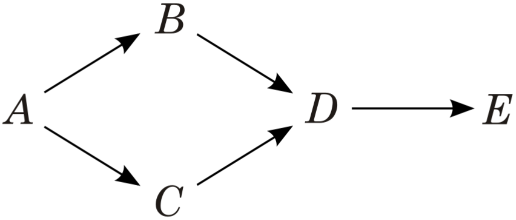

Definition (d-connection condition)

A causal model satisfies the d-connection condition if and only if for all $X, Y \in V$ and $Z \subseteq V \backslash \{X, Y\}$: If $Dep(X, Y|Z)$, then X and Y are d-connected by Z.

Causal Bayes nets

Definition (causal Markov condition)

A causal model satisfies the causal Markov condition (CMC) if and only if every X is probabilistically independent of its non-effects conditional on its direct causes. (cf. Spirtes et al., 2000, p. 29)

CMC determines the following Markov factorization:

$$P(x_1, ..., x_n) = \prod_{i=1}^{n} P(x_i | par(X_i)) \quad (1)$$

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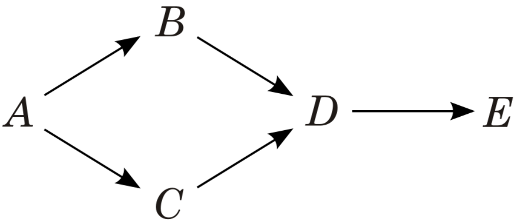

$$P(a, b, c, d, e) = P(a) \cdot P(b|a) \cdot P(c|a) \cdot P(d|b, c) \cdot P(e|d)$$

$Indep(B, C|A)$ $\quad$ $Indep(C, B|A)$

$Indep(D, A|\{B, C\})$ $\quad$ $Indep(E, \{A, B, C\}|D)$

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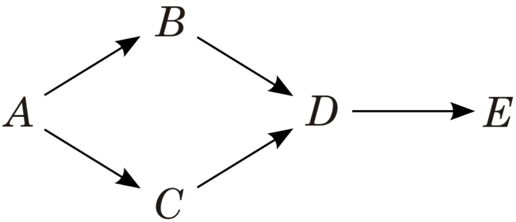

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Causal Bayes nets

The causal Markov condition is assumed to be satisfied by causal models that satisfy the causal sufficiency condition.

Definition (causal sufficiency condition)

A causal model satisfies the causal sufficiency condition if and only if every common cause C of every pair $X, Y \in V$ is in V or is fixed to a certain value c.

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Causal Bayes nets

A causal model that satisfies CMC satisfies the causal faithfulness condition (CFC) if and only if the independencies implied by CMC are all the independencies in the model (cf. Spirtes et al., 2000, p. 31).

Generalized:

Definition (causal faithfulness condition)

A causal model satisfies the causal faithfulness condition if and only if every *d*-connection implies a probabilistic dependence. (cf. Schurz & Gebharter, 2015, sec. 3.2)

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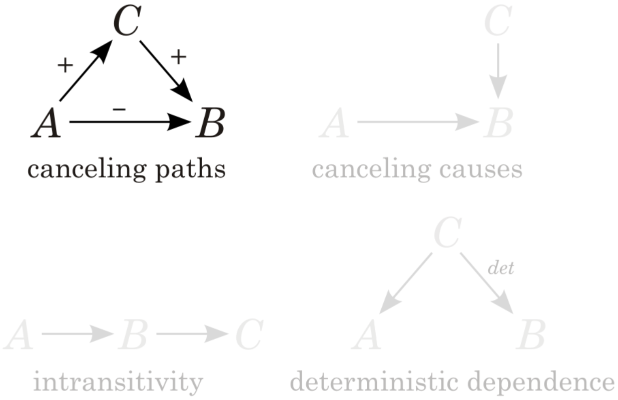

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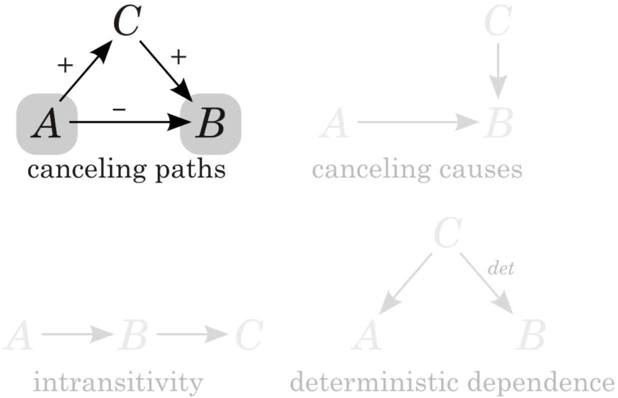

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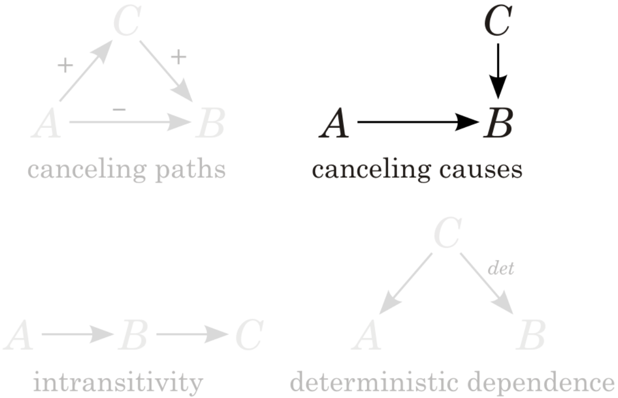

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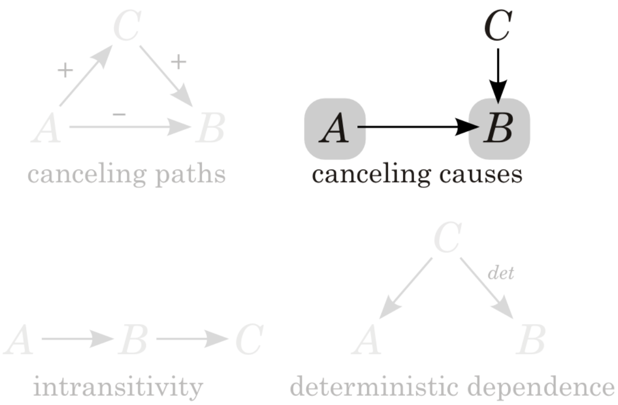

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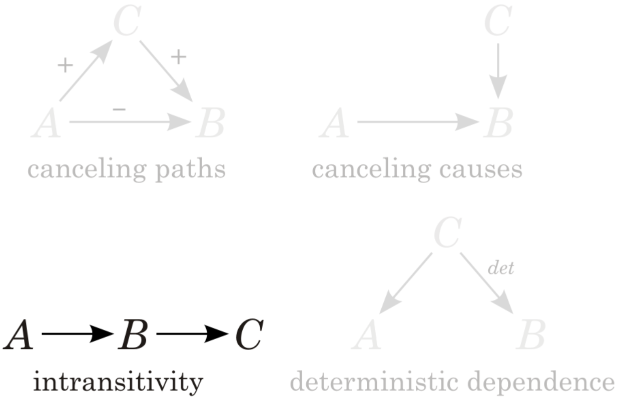

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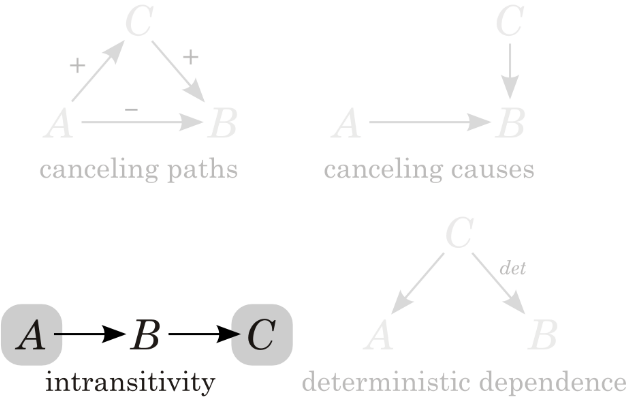

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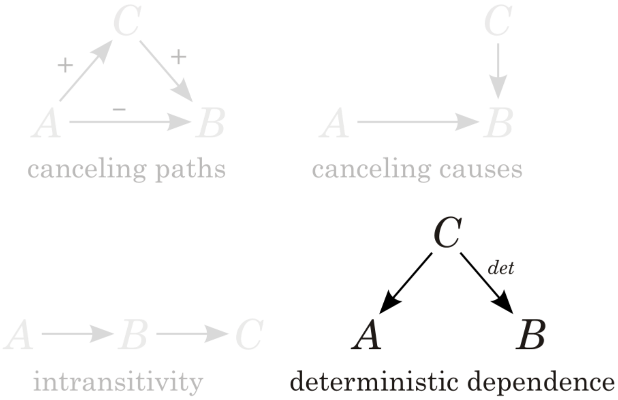

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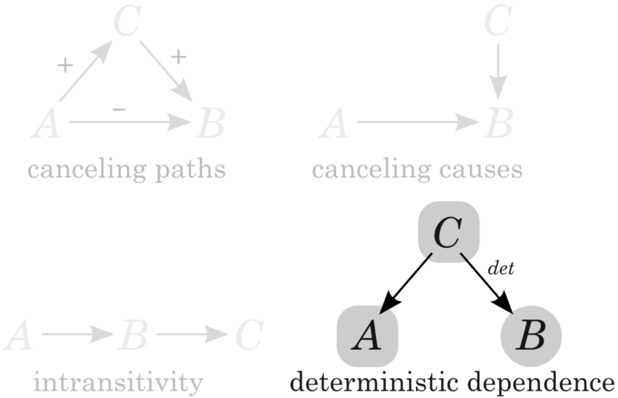

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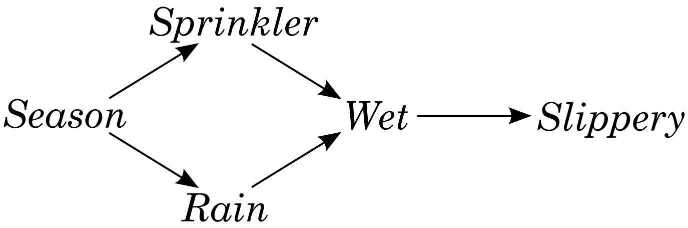
Intervention and observation

CBNs allow for distinguishing intervention from observation (cf. Pearl, 2009, sec. 1.3.1; Spirtes et al., 2000, sec. 3.7.2).



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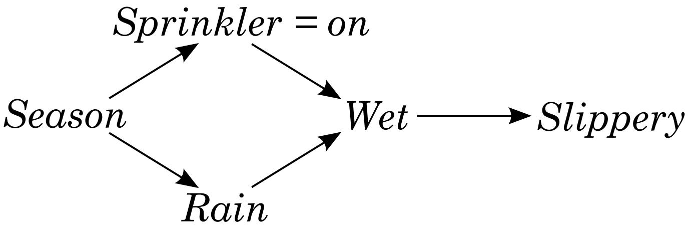

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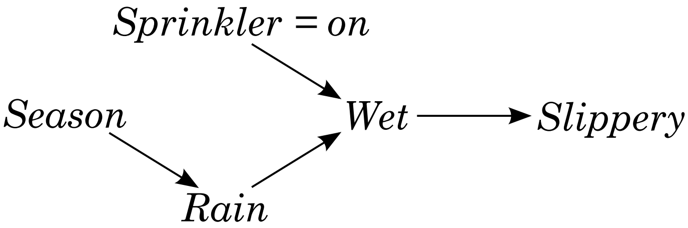

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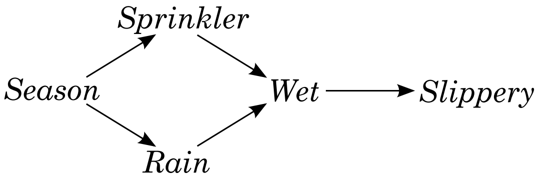
Causal reasoning with causal Bayes nets



Observation:

$$P(sl_1|sp_{on}) = \frac{P(sl_1, sp_{on})}{P(sp_{on})}$$

$$P(sl, sp_{on}) = \sum_{u} P(sl_1, sp_{on}, u), \textit{ where } U = V \backslash \{Sl, Sp\}$$

$$\sum_{u} P(sl_1, sp_{on}, u) = \sum_{se,ra,we} P(se) \cdot P(sp_{on}|se) \cdot P(ra|se) \cdot P(we|sp_{on}, ra) \cdot P(sl_1|we)$$

$$P(sp_{on}) = \sum_{w} P(sp_{on}, w), \textit{ where } W = V \backslash \{Sp\}$$

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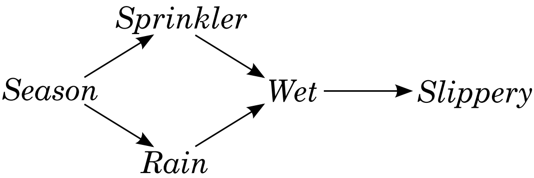

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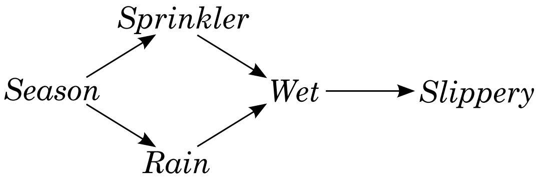

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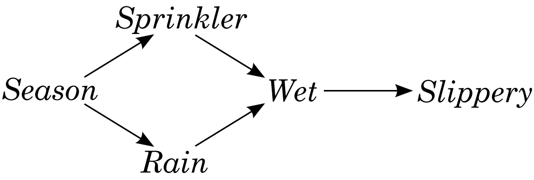
Causal reasoning with causal Bayes nets



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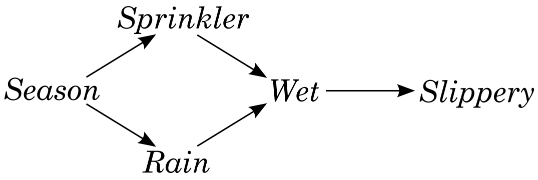

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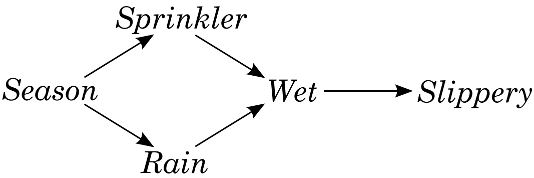

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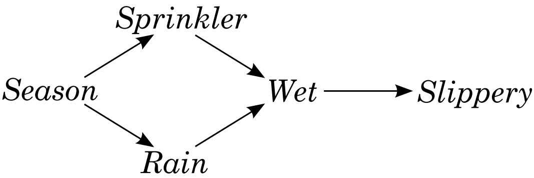

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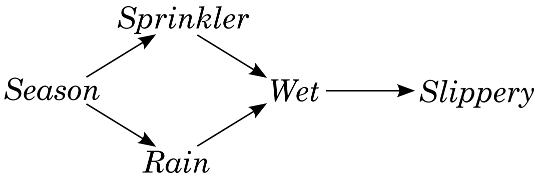

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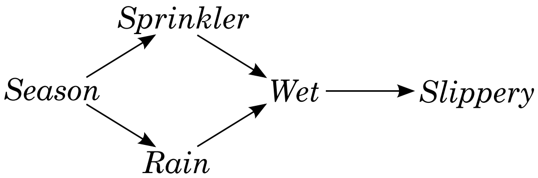

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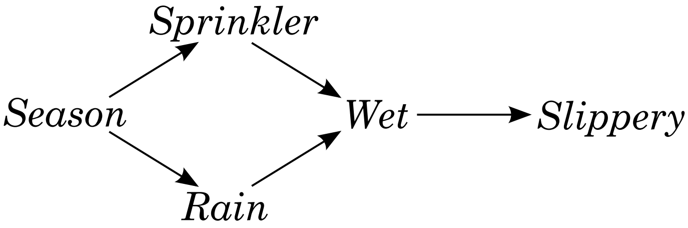

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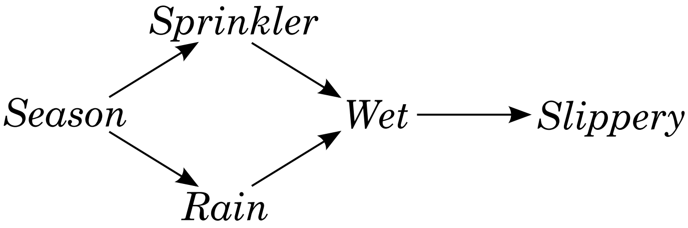

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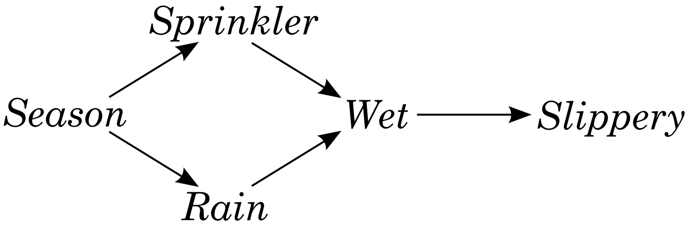

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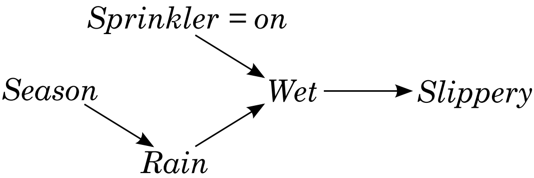

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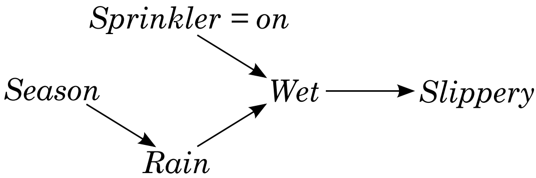

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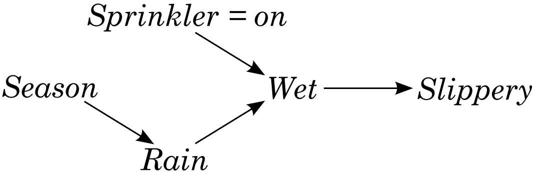

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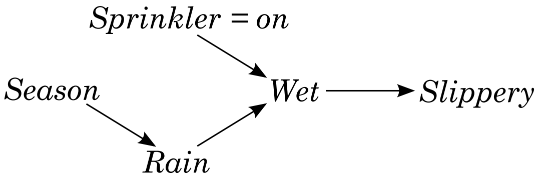

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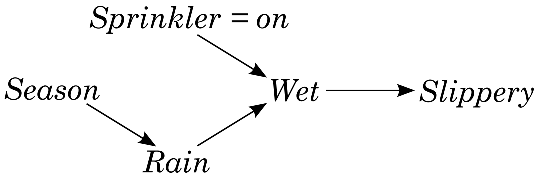

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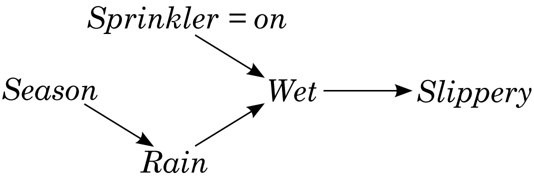

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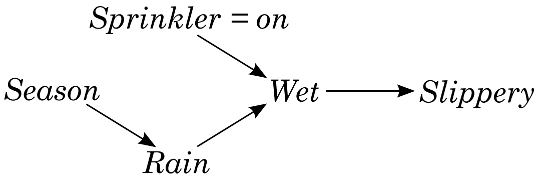

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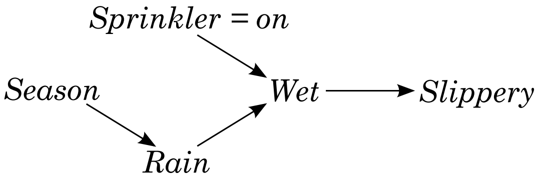

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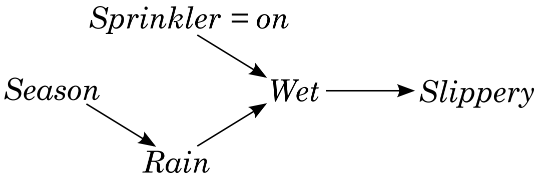

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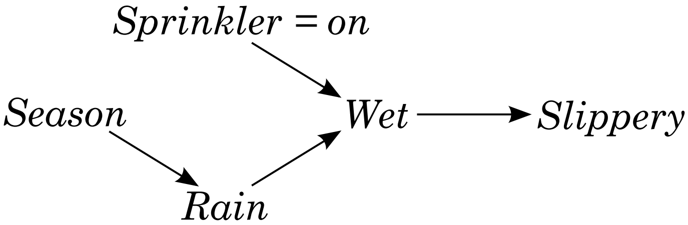

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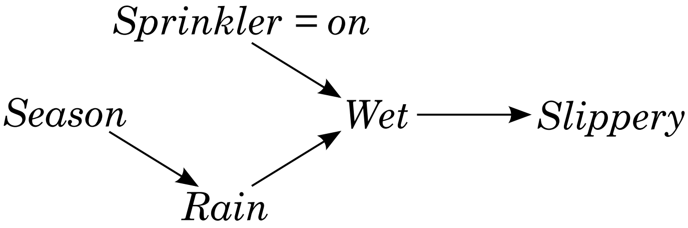

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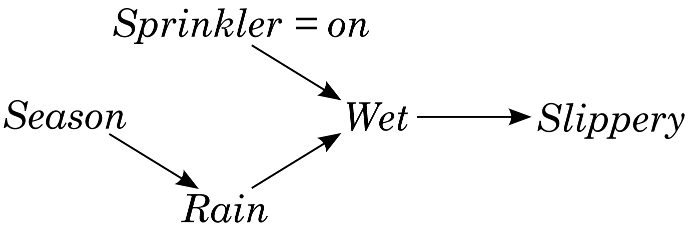

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SGS algorithm (cf. Spirtes et al., 2000, p. 82)

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S4: *(a) For all $X \longrightarrow Z — Y$ without an edge between X and Y: Orient $Z — Y$ as $Z \longrightarrow Y$.*
(b) If $X \longrightarrow ... \longrightarrow Y$ and $X — Y$, then orient $X — Y$ as $X \longrightarrow Y$.

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Causal discovery

SGS algorithm (cf. Spirtes et al., 2000, p. 82)

S1: *Form the complete undirected graph over vertex set V.*

S2: *Check for every $X — Y$ for which there is a $Z \subseteq V \backslash \{X, Y\}$ such that $Indep(X, Y|Z)$, remove the edge between X and Y.*

S3: *For all $X — Z — Y$ (or $X \longrightarrow Z — Y$) without an edge between X and Y: Orient the edges as $X \longrightarrow Z \longleftarrow Y$ iff $Dep(X, Y|M)$ holds for all $M \subseteq V \backslash \{X, Y\}$ with $Z \in M$.*

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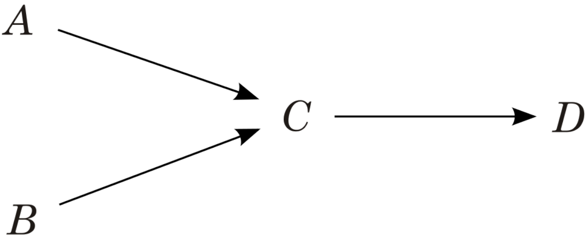
Causal discovery

Step 1 | Step 2 | Step 3 | Step 4



A & B: $Indep(A, B)$

A & D: $Indep(A, D|C)$ $Indep(A, D|\{B, C\})$

B & D: $Indep(B, D|C)$ $Indep(B, D|\{A, C\})$

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Causal discovery

Step 1 | Step 2 | Step 3 | Step 4

A

C D

B

A & B: $Indep(A, B)$

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Causal discovery

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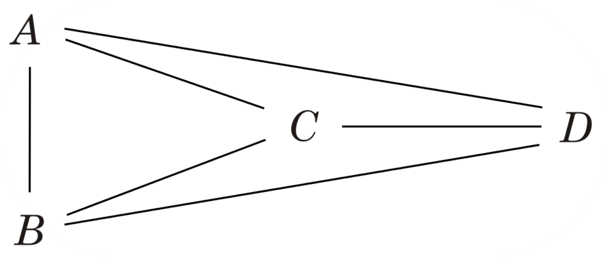

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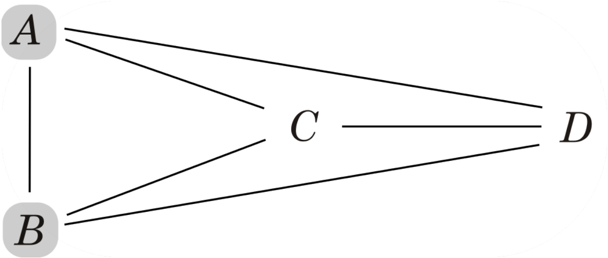
Causal discovery

Step 1 | Step 2 | Step 3 | Step 4



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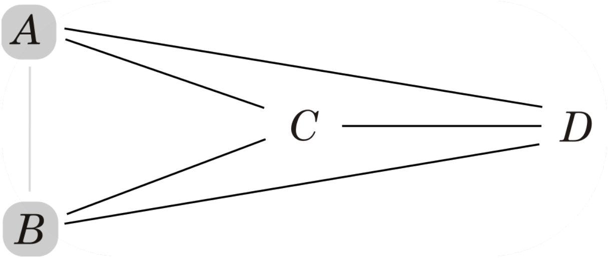
Causal discovery

Step 1 | Step 2 | Step 3 | Step 4



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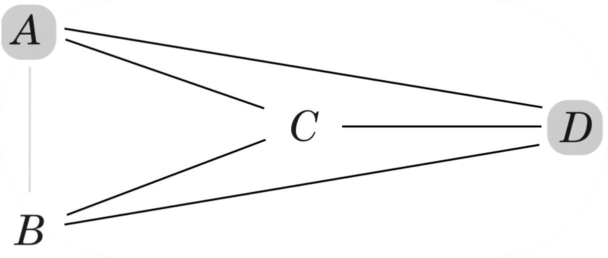
Causal discovery

Step 1 | Step 2 | Step 3 | Step 4



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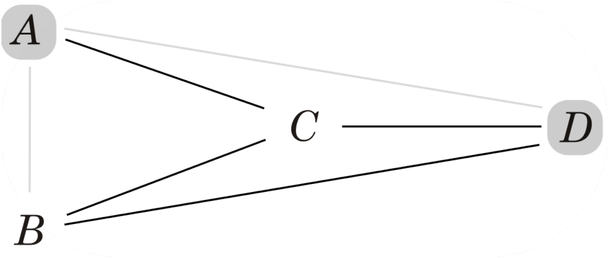

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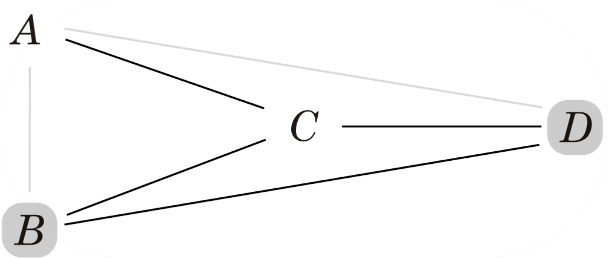

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Causal discovery

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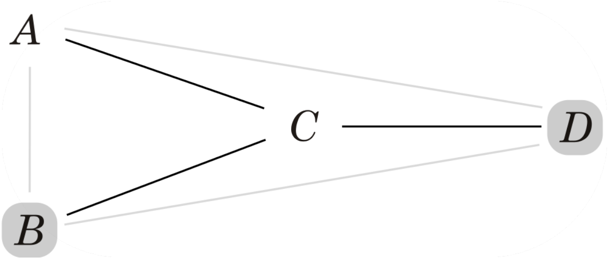

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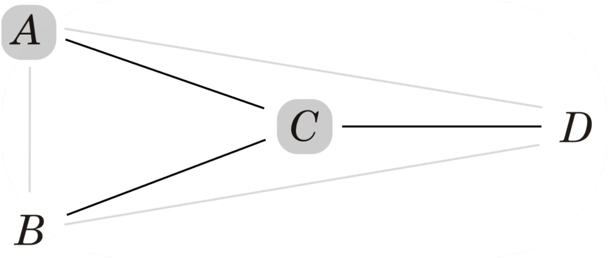

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Causal discovery

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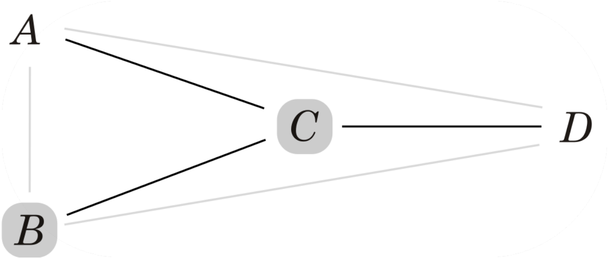

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Causal discovery

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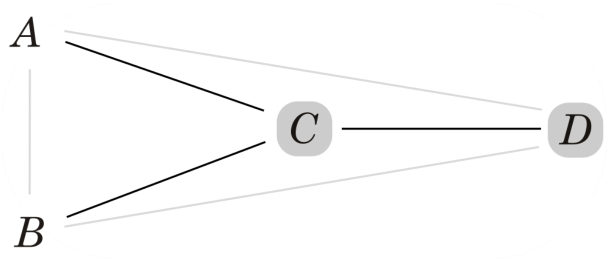

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Causal discovery

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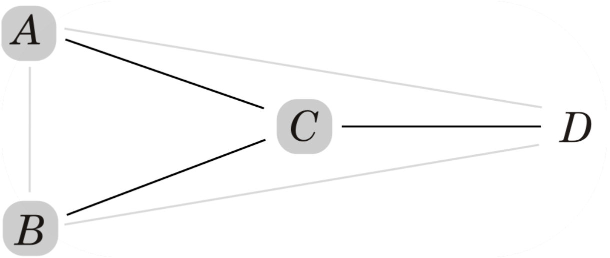

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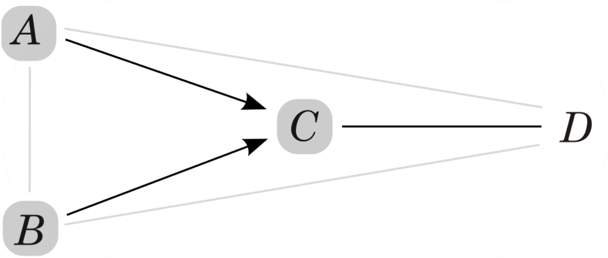

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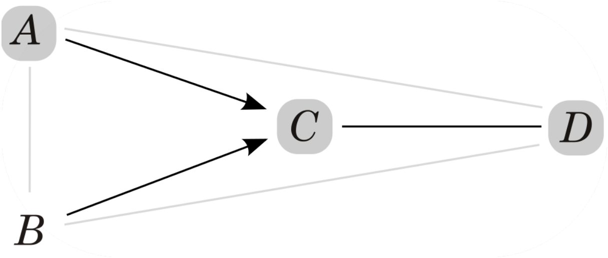

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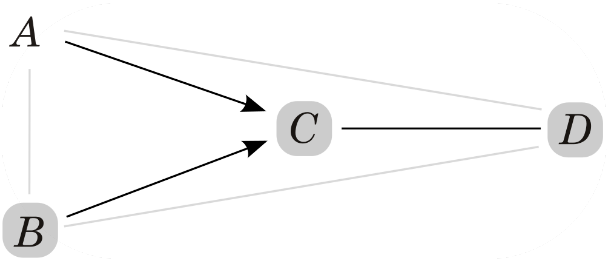

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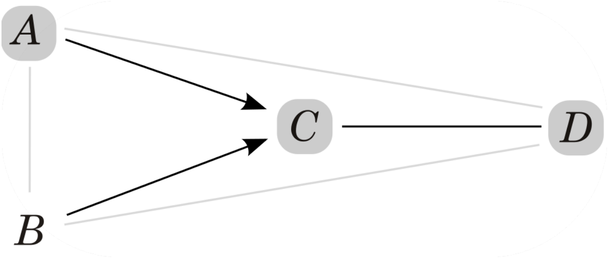

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Causal discovery

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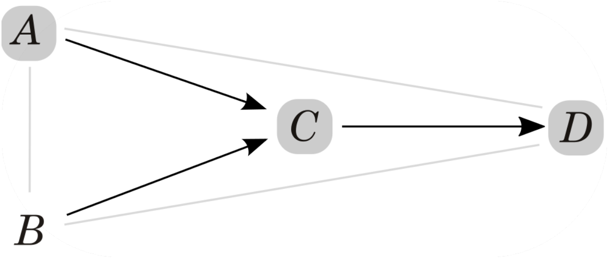

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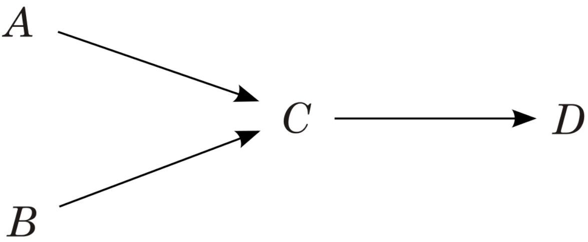

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Many thanks!

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