How Does the World Compute?

Embodied and disembodied computing at the Turing Centenary

S. Barry Cooper - SGSLPS one-day conference on *COMPUTABILITY AND LOGIC*, Bern, Switzerland, 5 April, 2012

Alan Turing 1912-1954

By sculptor Glyn Hughes, unveiled June 23, 2001

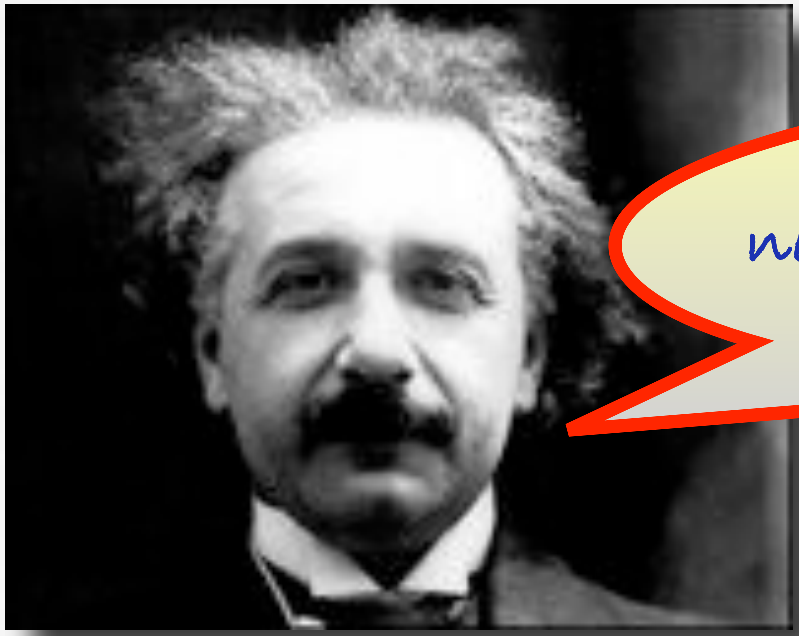
Former home of Turing, Holly Mead, Adlington Road, Wilmslow

Pushing Back the Incomputable - Ten Big Ideas

- The Turing Machine
- Programs as Data
- Embodied Computing
- Cryptography & Enigma
- Unsolvable Problems
- Exploring Incomputability
- Computing with Oracles
- Modelling the Brain
- The Turing Test & AI
- Morphogenesis

In Pursuit of the Computable

- Newton onwards - mathematics rules science
  - we look for computable natural laws ...
  - theories which computably predict ...
  - try to capture truth via proofs ...

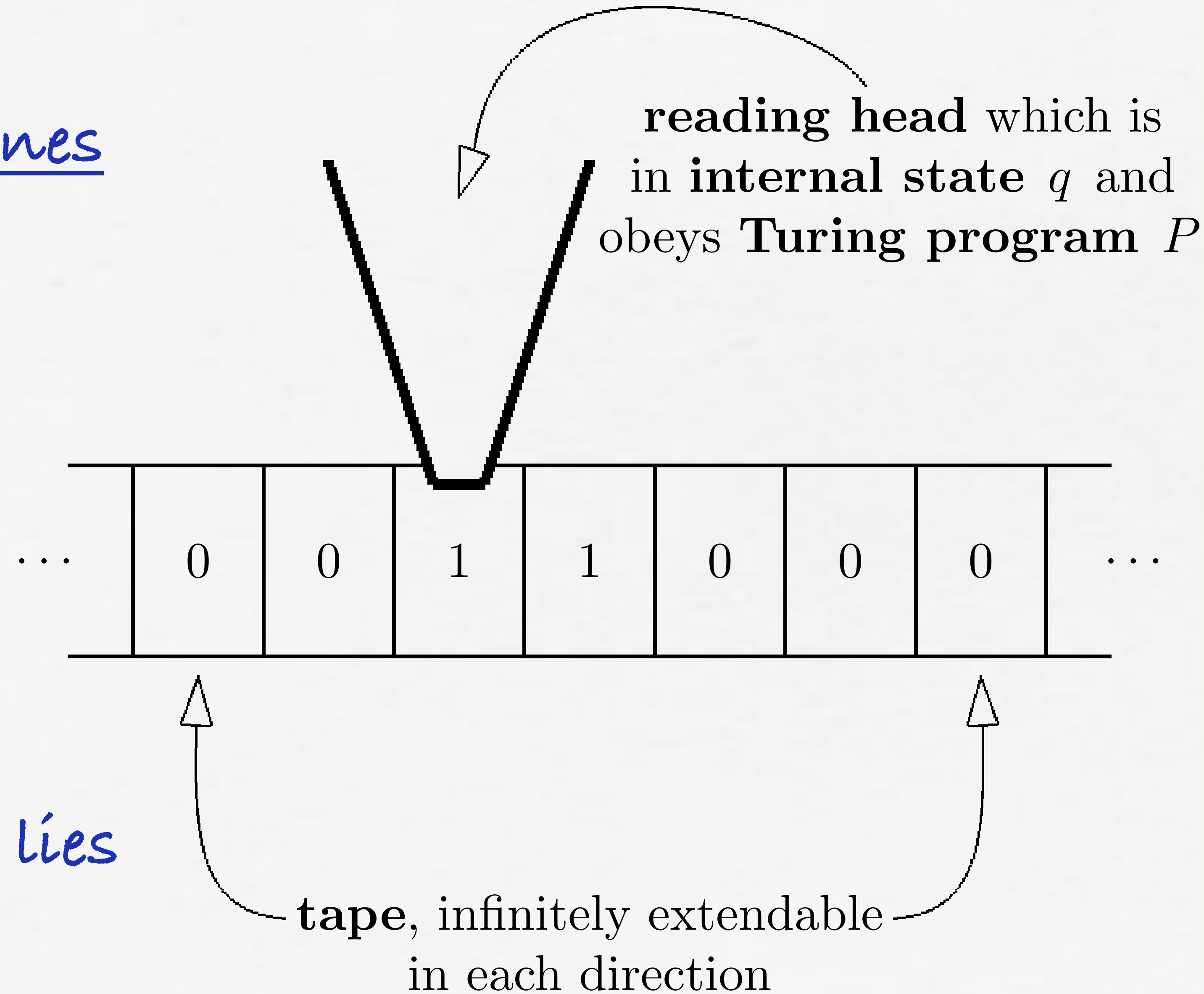


When we say that we understand a group of natural phenomena, we mean that we have found a constructive theory which embraces them

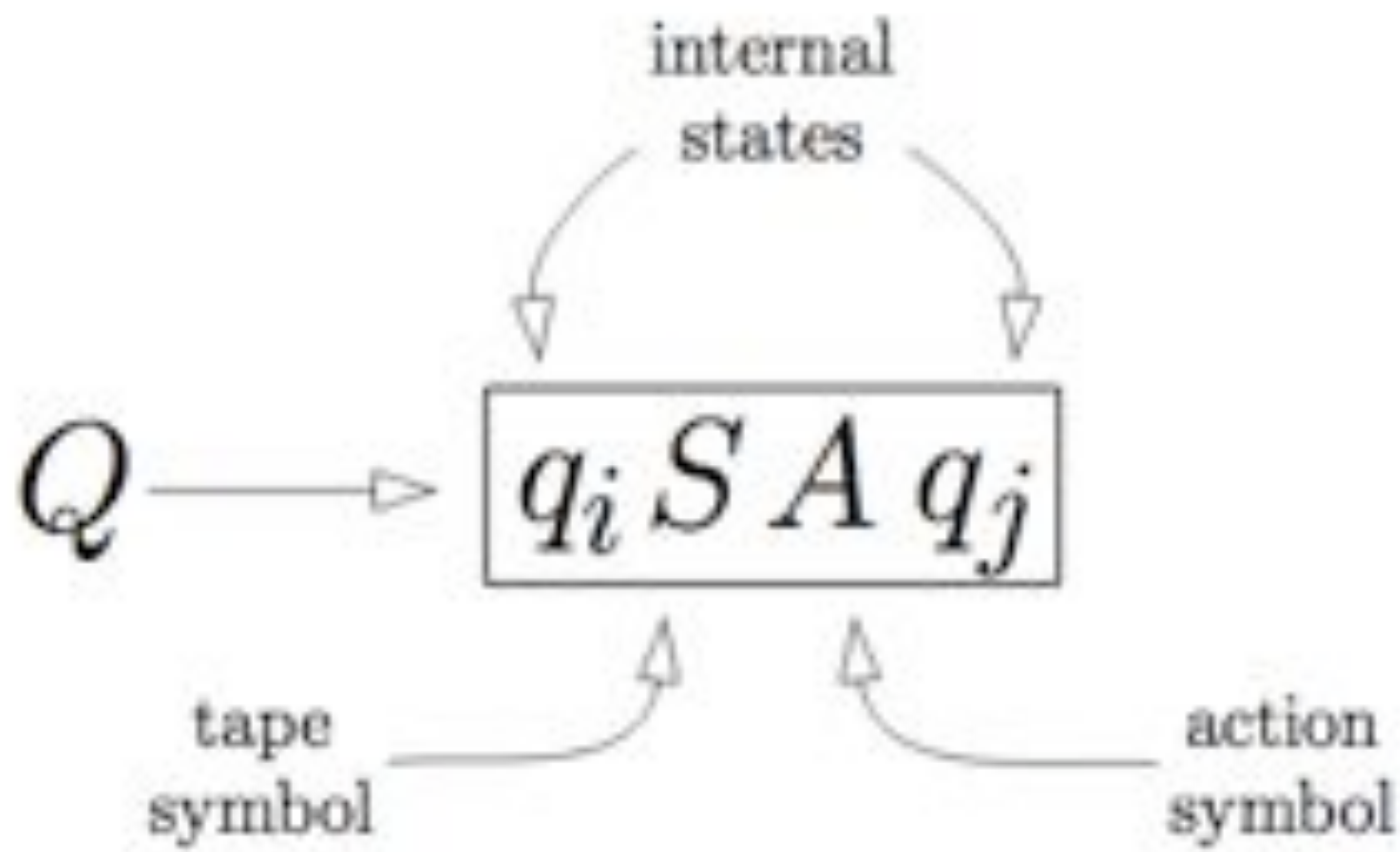
Albert Einstein: P.54, `Out of My Later Years', 1950

1. Computation Disembodied

- 1936 - Turing's machines
- Hardware trivial
- Actions simple
- But compute 'anything' computable
- All the computing power lies in the program ...



The program for T will be formed from instructions — called *quadruples* — made up from the programming symbols. Any such quadruple Q must take the form:



Then $Q = q_i S A q_j$ says: "If T is in state q_i reading tape symbol S, then perform action A and pass into new internal state q_j." If T is in state q_i reading tape symbol S, then we say that Q is *applicable*.

What does this Turing machine compute?

$q_0 1 0 q_1$	$\leftarrow$	subroutine for deleting 1's
$q_1 0 R q_0$	$\leftarrow$	move right in search of another 1, in preparation for return to "delete" subroutine

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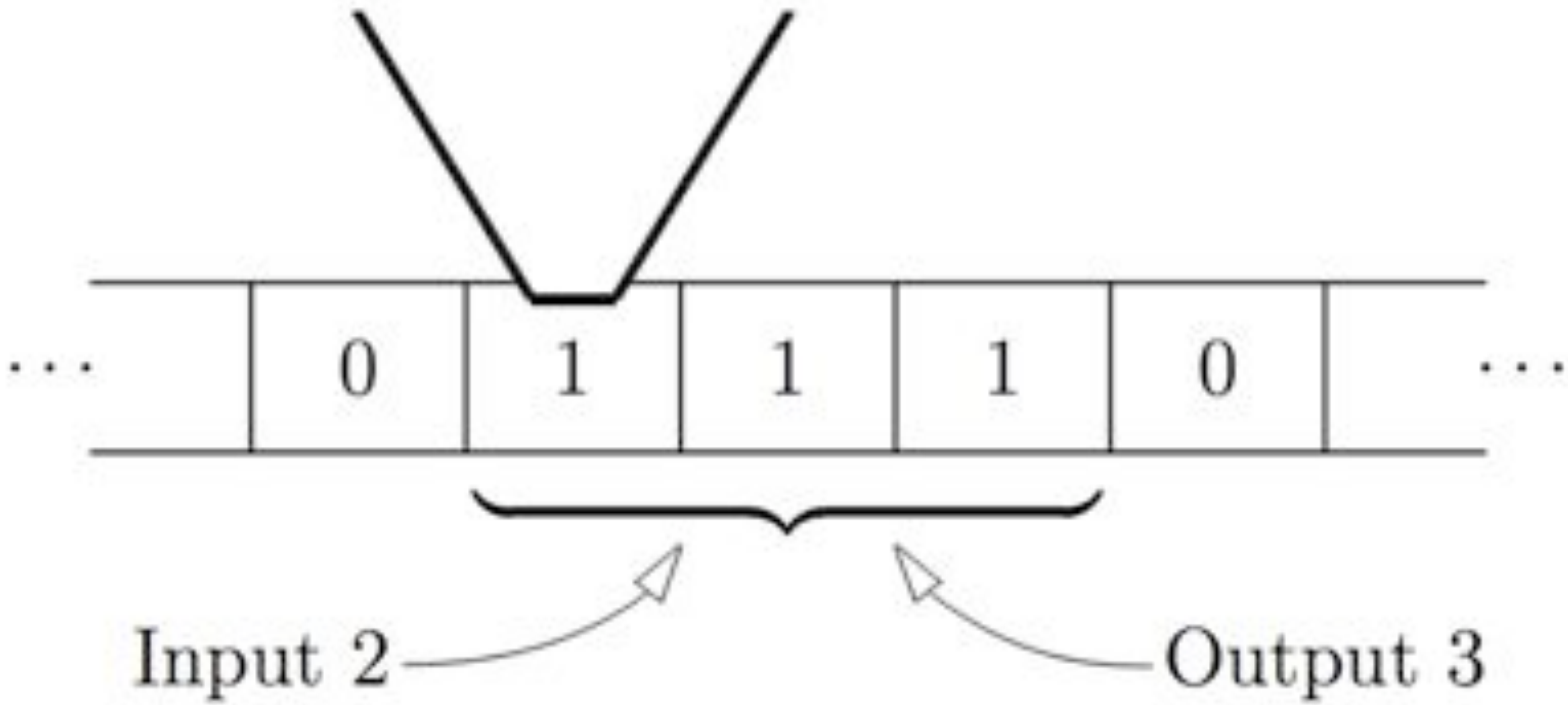

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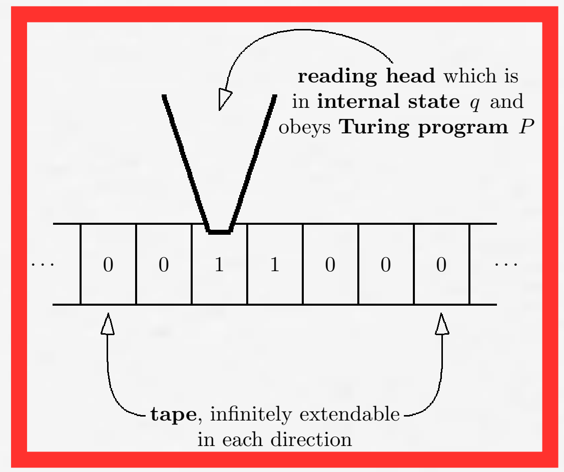
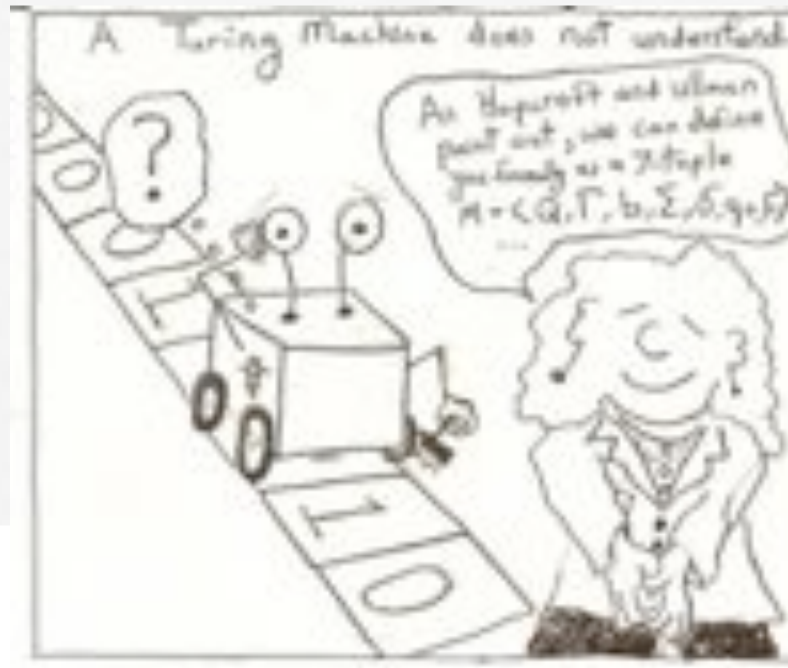
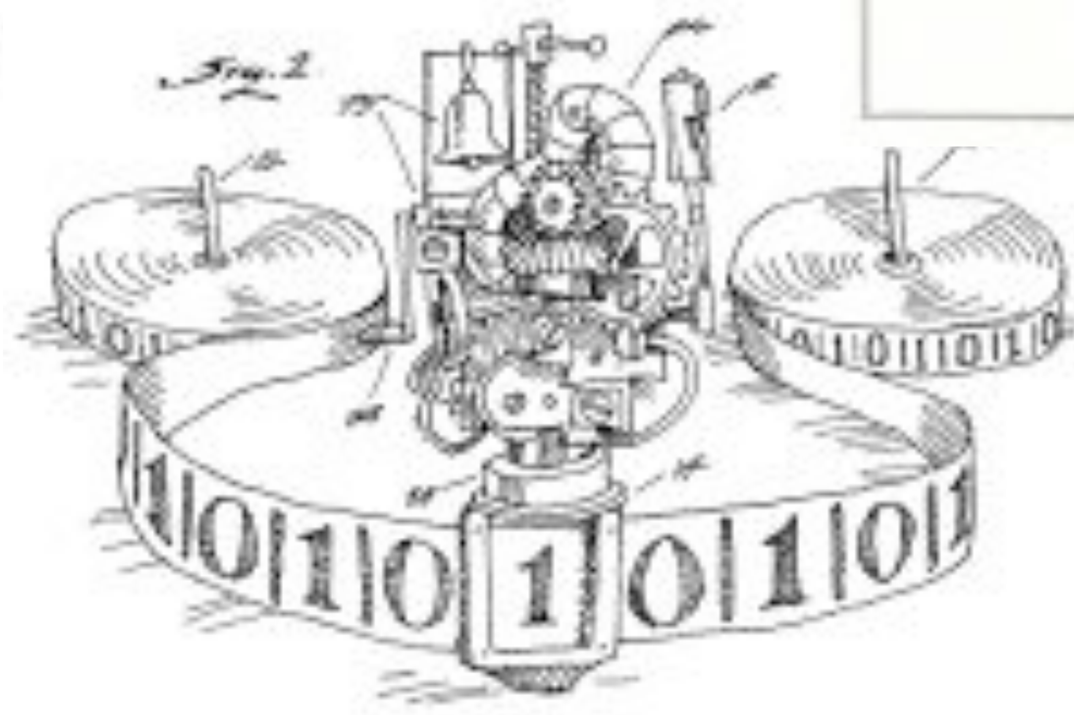
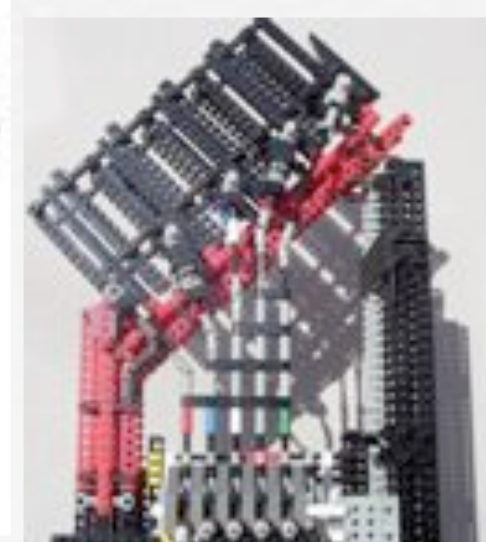
$$\varphi_T(n) = 0 = \mathbf{0}(n) \text{ for all } n \in \mathbb{N}.$$

1. Computation Disembodied

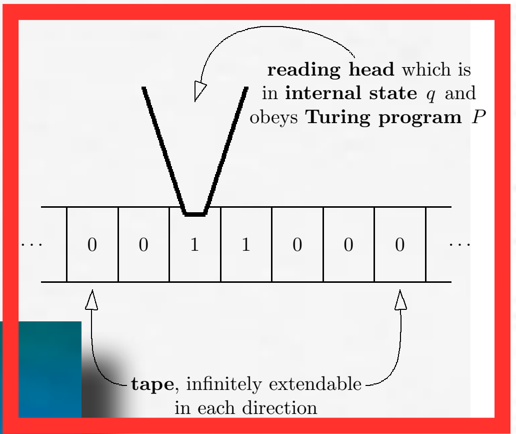


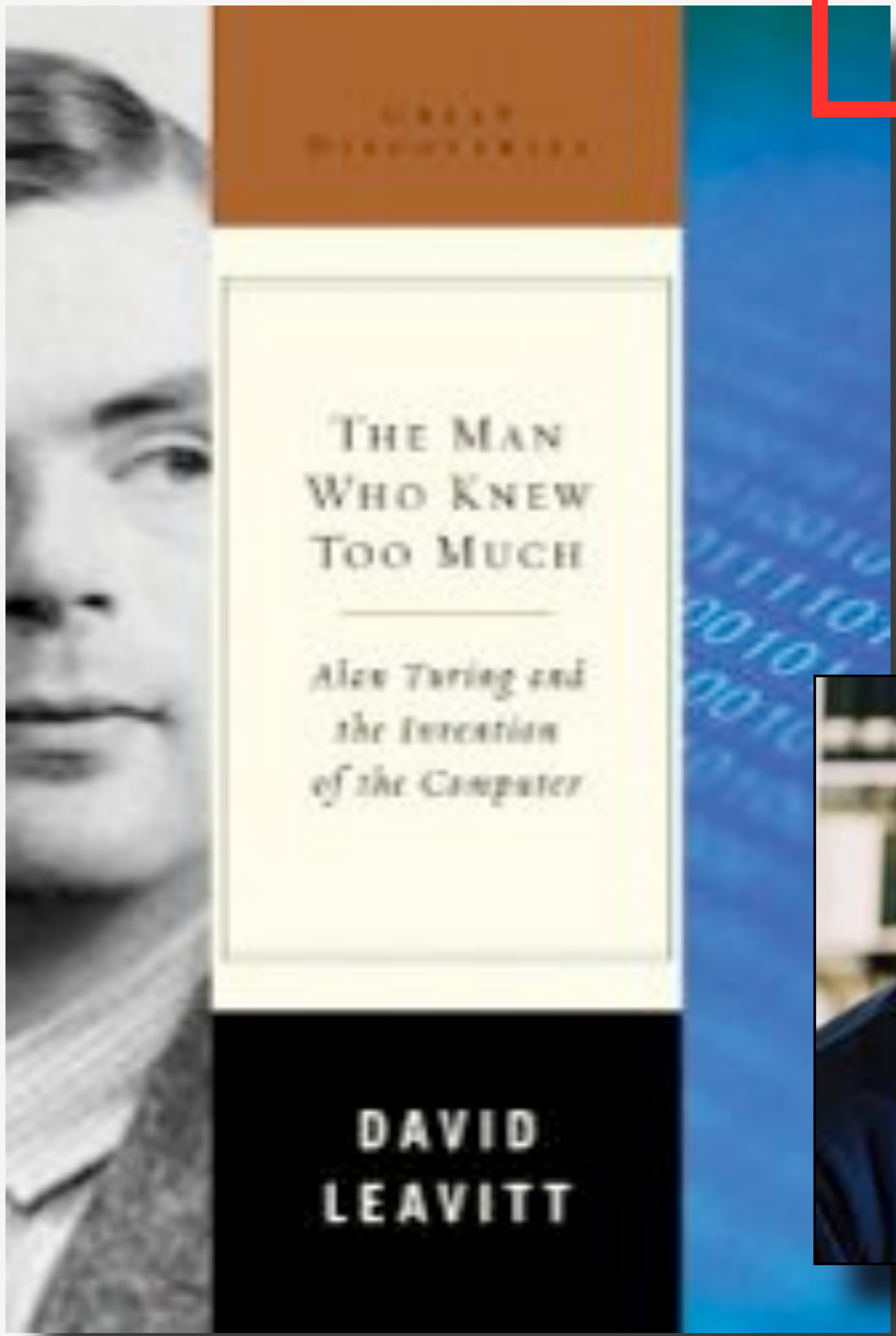






1. Computation Disembodied





2. Universality, and Programs as Data

Turing, 1936: Build a UNIVERSAL TURING MACHINE, which can simulate ANY other machine

Can computably code, and so list, the Turing programs, giving:

$$\varphi_e^{(k)} = \text{ the } k\text{-place partial function computed by } P_e.$$

$$\varphi_e = \varphi_e^{(1)} \ \textit{the}\ \mathrm{e}^{\mathrm{th}}\ \textbf{partial computable (p.c.) function}$$

2. Universality, and Programs as Data

Turing, 1936: Build a UNIVERSAL TURING MACHINE, which can simulate ANY other machine

There exists a Turing machine U — the **Universal Turing Machine** *— which if given input* (e, x) *simulates the* e^{th} *Turing machine with input* x.

That is, $\varphi_U^{(2)}(e, x) = \varphi_e(x)$.

2. Universality, and Programs as Data

- Turing, 1936: Build a UNIVERSAL TURING MACHINE, which can simulate ANY other machine
- An anticipation of the stored program computer
- A concept immediately understood by John von Neumann - as in his 1945 EDVAC report
- And 'program as data' key to the first computer …

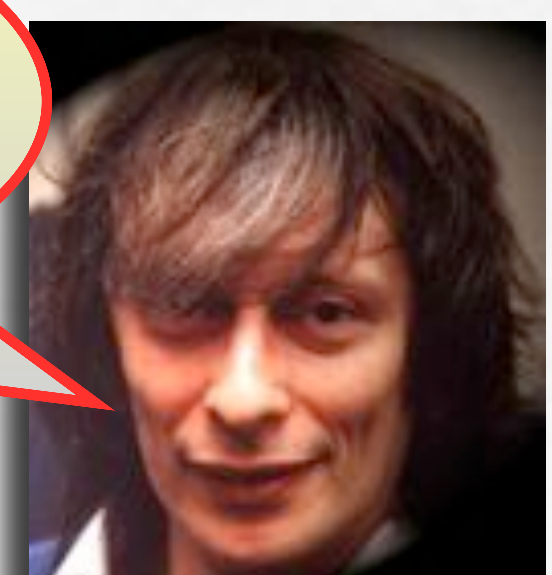
A New Computing Paradigm ...

- The omnipotent computer – I am building a brain
- Functionalism and AI – stress what a computer does as something realisable in different hardware – Hilary Putnam: "Minds and Machines", 1960
- Virtual Machine (IBM, 1965)– software implementation of a programmable machine – JAVA, Unix

A New Computing Paradigm ...

- Successful reduction of "natural" examples to the Turing model – e.g. quantum computation (David Deutsch)

I am sure
we will have [conscious computers], I expect they will be purely classical, and I expect that it will be a long time in the future. Significant advances in our philosophical understanding of what consciousness is, will be needed.



Question and Answers with David Deutsch, on New.Scientist.com News Service, December, 2006

A New Computing Paradigm …

- Martin Davis versus the hypercomputationalists (Jack Copeland et al) -

> The great success of modern computers as all-purpose algorithm-executing engines embodying Turing's universal computer in physical form, makes it extremely plausible that the abstract theory of computability gives the correct answer to the question 'What is a computation?', and, by itself, makes the existence of any more general form of computation extremely doubtful.

Martin Davis [2004], *The myth of hypercomputation*. In *Alan Turing: Life and legacy of a great thinker* (C. Teuscher, ed.), Springer-Verlag

Articles

UBIQUITY SYMPOSIUM 'WHAT IS COMPUTATION?' COMPUTATION IS PROCESS

November 2010 | BY DENNIS J. FRAILEY

Full text also available in the ACM Digital Library as PDF | HTML

Printer Friendly Version PDF

Various authors define forms of computation as specialized types of processes. As the scope of computation widens, the range of such specialties increases. Dennis J. Frailey posits that the essence of computation can be found in any form of process, hence the title and the thesis of this paper in the Ubiquity symposium discussion what is computation. –Editor

The concept of computation is arguably the most dramatic advance in mathematical thinking of the past century. Denning [2010], in his opening statement, describes how computation was originally defined in the 1930s and how that definition has progressed through the ensuing decades. Church, Gödel, and Turing defined it in terms of mathematical functions, which they divided into the decidable (can be evaluated by algorithms) and the un-

"What Is Computation?"

\- <u>ACM *UBIQUITY* SYMPOSIUM</u>: *Computation Is Process* by Dennis J Frailey:

- The <u>concept of computation</u> is arguably the most dramatic advance in mathematical thinking of the past century

- <u>Church, Gödel, and Turing</u> defined it in terms of <u>mathematical functions</u> ... They were inclined to the view that only the algorithmic functions constituted computation

- I'll call this the "<u>mathematician's bias</u>" because I believe it limits our thinking and prevent us from fully appreciating the power of computation

Program-Data Duality ...

Re 'what we compute',
Turing took traditional mathematical objects, real numbers, functions etc. as the things to be computed. In subsequent work in Computer Science, the view of computation has broadened enormously. In the work on concurrent processes, the behaviour *is* the object of interest. There is indeed a lack of a clear-cut Church-Turing thesis in this wider sphere of computation - computation as interaction, as Robin Milner put it.

Samson Abramsky, private communication, March 2011

Program-Data Duality ...

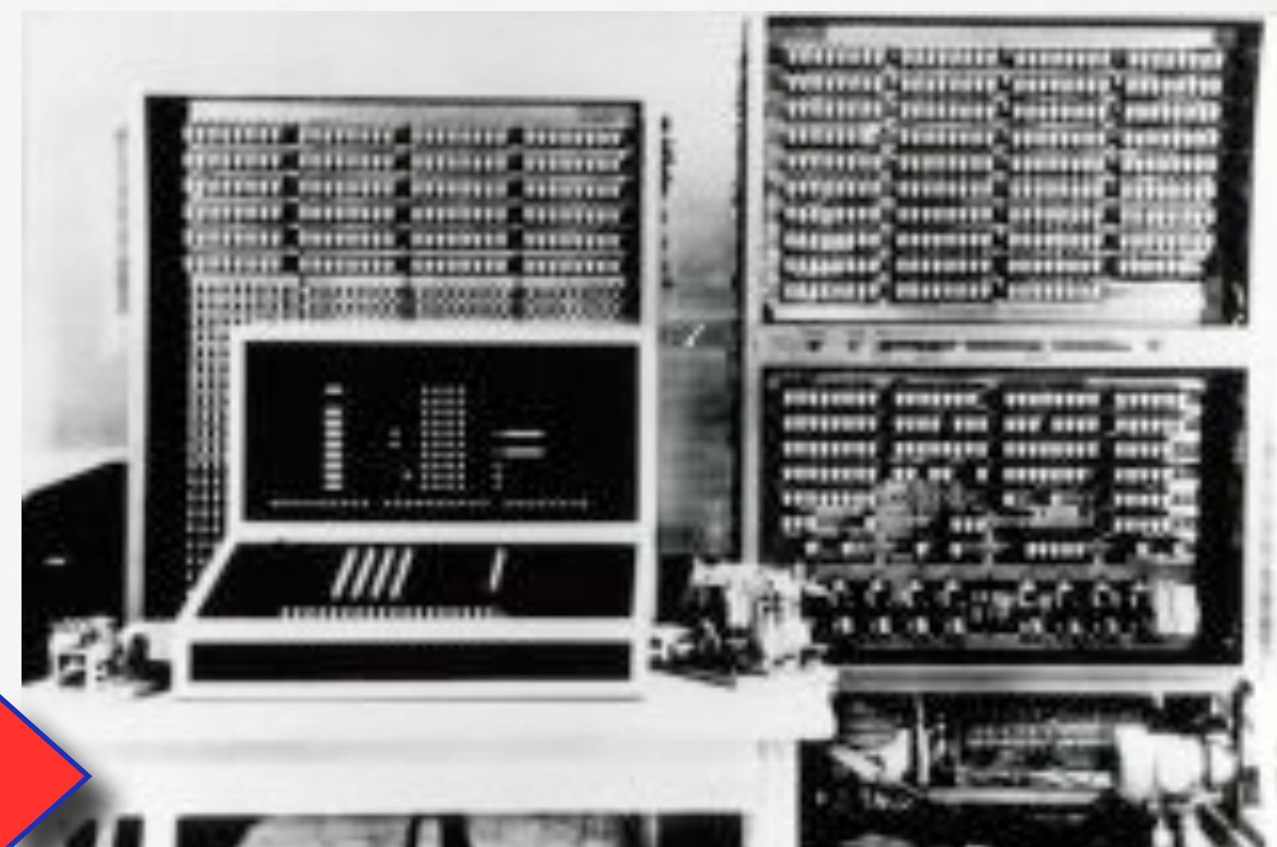
- "Formally, giving a program + data logically implies the output (leaving aside non-determinism or randomness), so why actually bother computing the result!

- "... Can information increase in computation? Information theory and thermodynamics seem to tell us that it can't, yet intuitively, this is surely exactly why we compute - to get information we didn't have before."

3. Programs as Data Embodied

Konrad Zuse

- The first electronic digital computer - John Atanasoff (1937-42)?
- First stored program computer that worked - Manchester ‘Baby’ (1948)
- First commercial computer - EDSAC in Cambridge (Maurice Wilkes, 1949), or Eckert and Mauchly’s UNIVAC (1951)
- If ‘Program as data’ key to the first computer … out go Babbage; Zuse, 1930s; Colossus,1944; ENIAC,1946

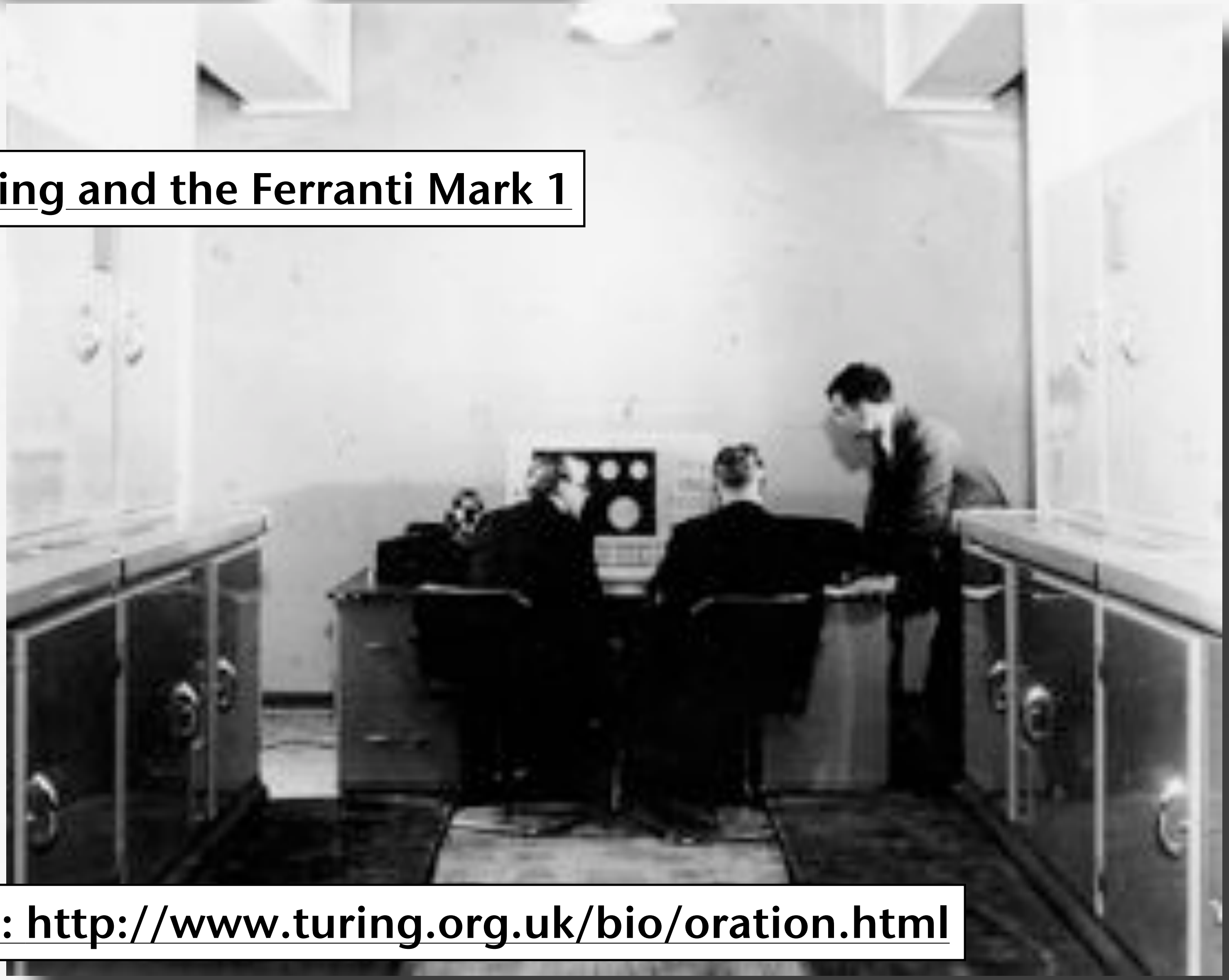


3. Programs as Data Embodied

Turing Bombe Rebuild at Bletchley Park

Pilot ACE, May 10, 1950 - small version of plan contained in Turing's ACE Report of 1945

3. Programs as Data Embodied



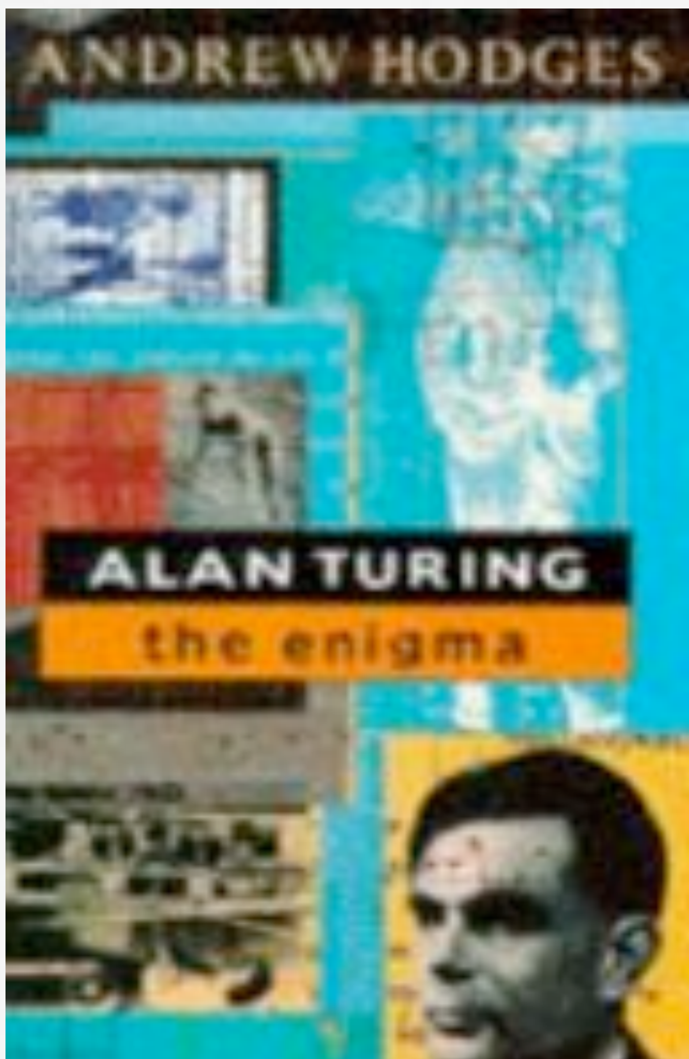
Turing and the Ferranti Mark 1

See: http://www.turing.org.uk/bio/oration.html

Dietrich Prinz and Computer Chess

4. Information - Hiding & Unhiding

Information



- ☐ Churchill: "the geese that laid the golden eggs but never cackled"

5. The Discovery of Unsolvability

Hilbert's Programme:

"For the mathematician there is no Ignorabimus, and, in my opinion, not at all for natural science either. ... The true reason why [no one] has succeeded in finding an unsolvable problem is, in my opinion, that there is no unsolvable problem.
In contrast to the foolish Ignorabimus, our credo avers:
We must know,
We shall know. "

- David Hilbert's opening address to the Society of German Scientists and Physicians, Königsberg, September 1930

5. The Discovery of Unsolvability

- Turing (1936): Take a <u>universal</u> Turing machine **U**:

<u>Unsolvability of the Halting Problem for U</u>:
No computer can tell us, for each given input x, whether U will compute - where, <u>remember</u> - we allow an input to include a coded program!

<u>'Church's Theorem'</u>:
No computer can tell us, for each given sentence, whether it is logically valid or not.

1944: Post's Problem

- February 26, 1944 - Post addresses the New York meeting of the American Mathematical Society
- May 1944 - publishes in the Bulletin of the AMS: "Recursively Enumerable Sets of Positive Integers and Their Decision Problems"
- QUESTION: Are there lots of natural examples of unsolvable problems?

What came later ...

Post's remark in this paper that Hilbert's tenth problem "begs for an unsolvability proof" had a major influence on my own work.

Martin Davis

From introduction to "Solvability, Provability, Definability: The Collected Works of Emil L. Post", Birkhauser, 1994

PROBLEM: **Given any polynomial equation in one or more variables, with integer coefficients, find a solution *consisting entirely of integers* — that is, solve any given *Diophantine equation*.**

HILBERT'S TENTH PROBLEM: **To find a general way of telling *effectively* whether a given Diophantine equation has a solution or not.**

A set $A \subseteq \mathbb{N}$ *is* **Diophantine** *if*

$$A = \{x \in \mathbb{N} \mid (\exists y_1, \ldots, y_n \in \mathbb{N})[p_A(x, y_1, \ldots, y_n) = 0]\}$$

for some polynomial $p_A(x, y_1, \ldots, y_n)$ *(with integer coefficients).*

$$A = \{1, 3, 4, 5, 7, 8, 9, 11, \dots\}$$

Then $A = \{x \in \mathbb{N} \mid x = y_1^2 - y_2^2, \text{ some } y_1, y_2 \in \mathbb{N}\}$. So A is Diophantine with $p_A(x, y_1, y_2) = y_1^2 - y_2^2 - x$.

DAVIS' STRATEGY: **Show that *every* computably enumerable set is Diophantine.**

But say -

$$K = \{x \in \mathbb{N} \mid (\exists y_1, \dots, y_n \in \mathbb{N})[p_K(x, y_1, \dots, y_n) = 0]\}$$

Known incomputable c.e. set

Then no algorithm exists for the equations:

$$p_K(0, y_1, \dots, y_n) = 0$$
$$p_K(1, y_1, \dots, y_n) = 0$$
$$p_K(2, y_1, \dots, y_n) = 0$$
$$\vdots$$

- Julia Robinson, Davis, Hilary Putnam (1960): For Davis' strategy, just need to show one exponentially increasing set is diophantine.

- Enter some more old mathematics: Fibonacci sequences are exponentially increasing, e.g., $\frac{1}{\sqrt{5}}[\frac{1}{2}(1+\sqrt{5})]^n$ approximates

$$1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots, a_n, a_{n+1}, a_{n+2} = a_{n+1} + a_n, \ldots.$$

- Yuri Matiasevich (1970): The Fibonacci sequence is diophantine.

COROLLARY **(Davis, Matiasevich, Putnam, Robinson)**
(i) Every computably enumerable set is Diophantine.
(ii) There is no positive solution to Hilbert's Tenth Problem.

$$u + w - \text{FIB}(2u) - 2 = 0$$

$$l - 2\text{FIB}(2u) - 2a - 1 = 0$$

$$l^2 - lz - z^2 - 1 = 0$$

$$g - bl^2 = 0$$

$$g^2 - gh - h^2 - 1 = 0$$

$$m - c(2h + g) - 3 = 0$$

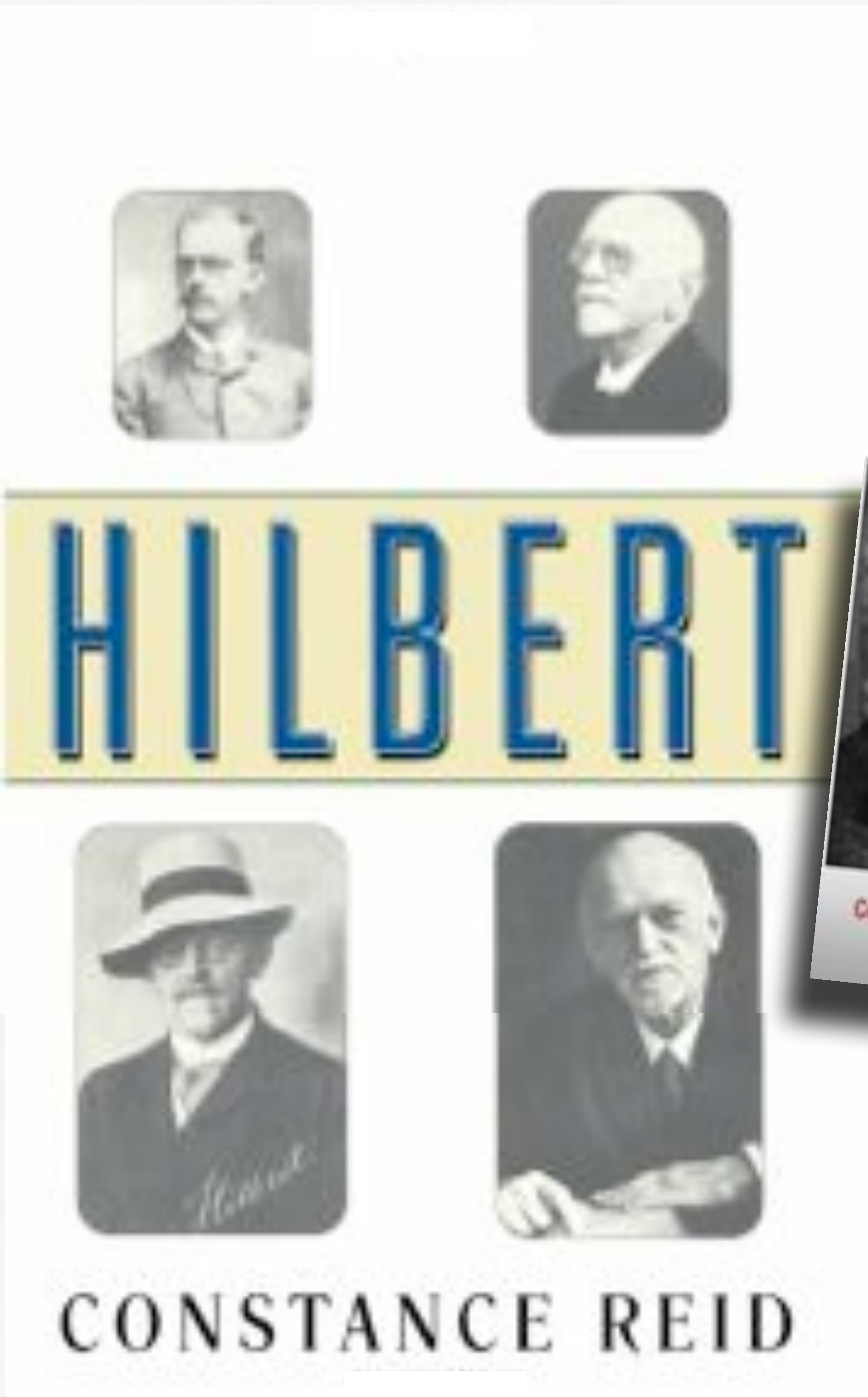
$$m - fl - 2 = 0$$

$$x^2 - mxy + y^2 - l = 0$$

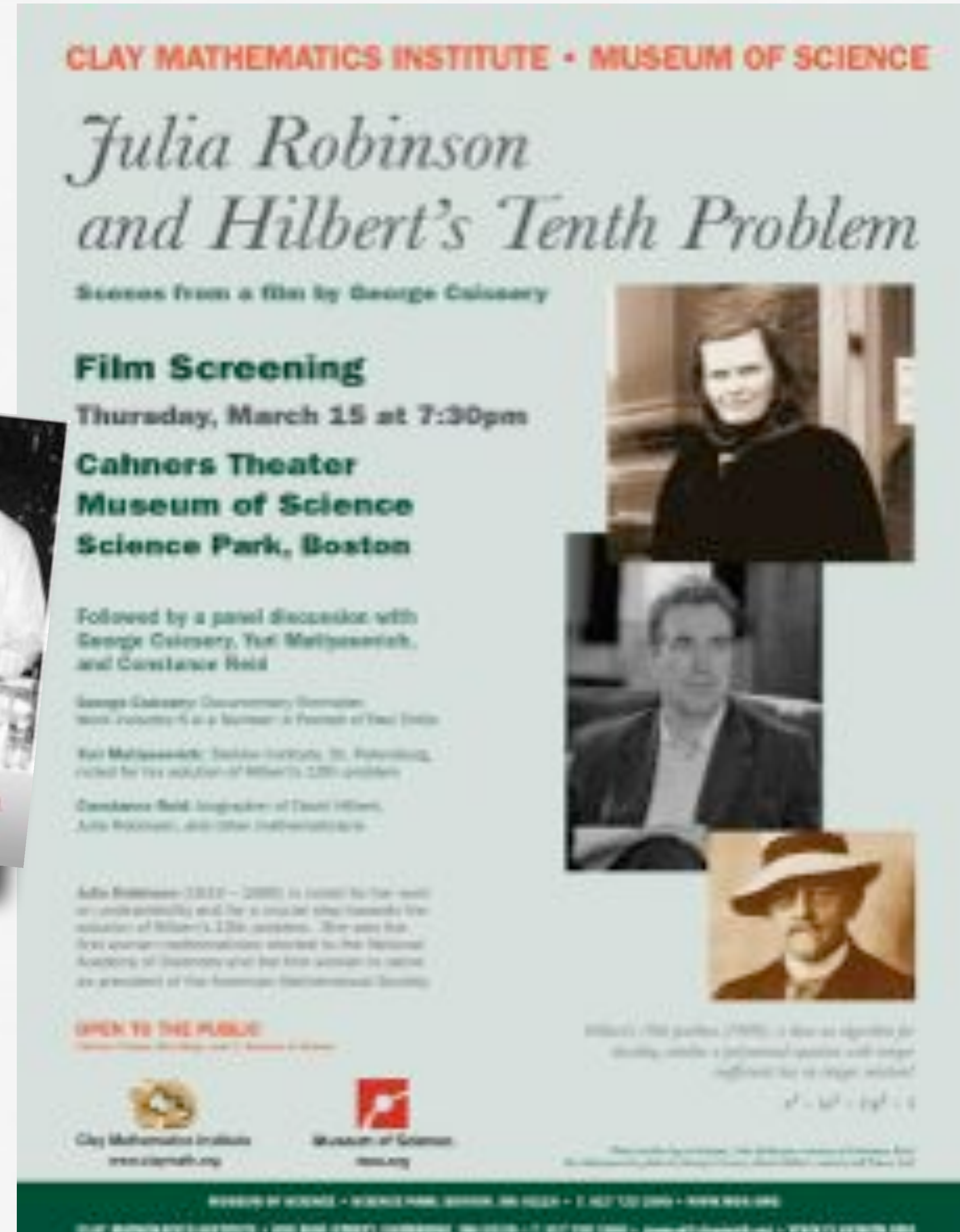
$$(d - l)l + u - x - 1 = 0$$

$$x - \text{FIB}(2u) - (2h + g)(l - 1) = 0.$$


Julia Robinson (1919 - 1985)


HILBERT
CONSTANCE REID


Constance Reid and Yuri Matiyasevich

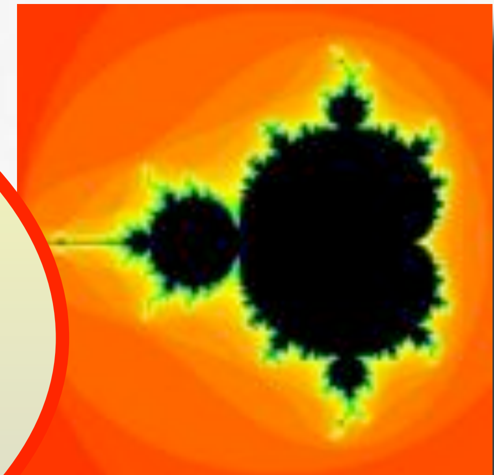

CLAY MATHEMATICS INSTITUTE • MUSEUM OF SCIENCE
Julia Robinson and Hilbert's Tenth Problem
Scenes from a film by George Csicsery
Film Screening
Thursday, March 15 at 7:30pm
Cahners Theater
Museum of Science
Science Park, Boston
Followed by a panel discussion with
George Csicsery, Yuri Matiyasevich,
and Constance Reid
OPEN TO THE PUBLIC

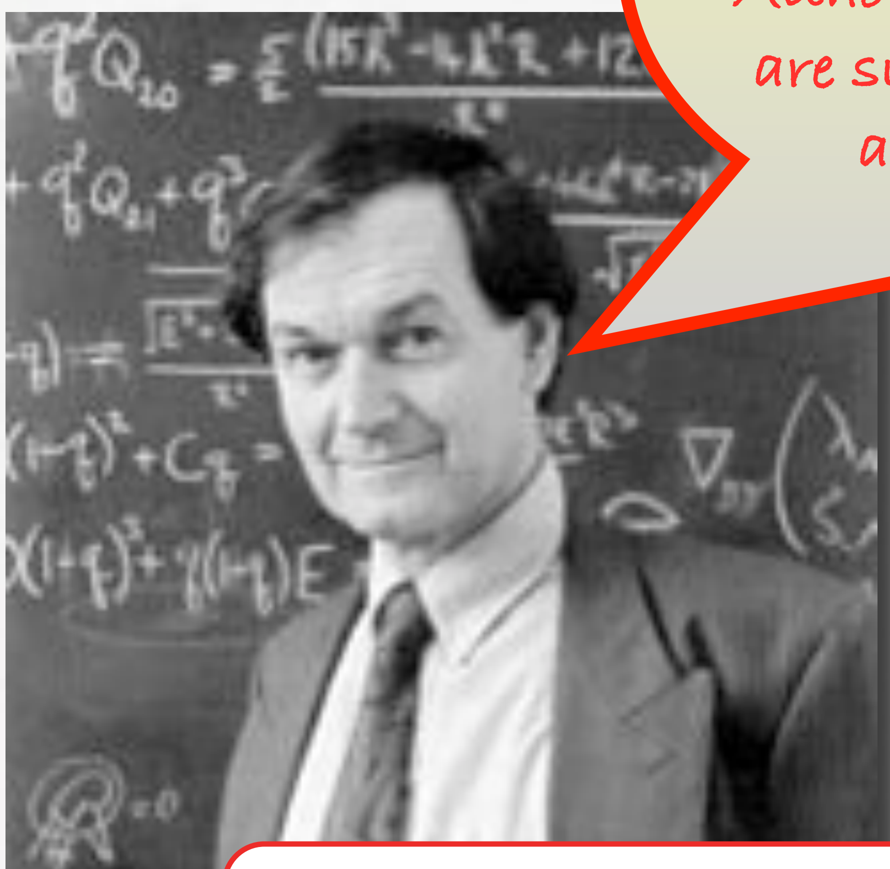
6. Map of Road to the Incomputable?

- Turing's observation: Things we cannot compute can often be approximated - as closely as we want!
- Maybe, using human ingenuity, we can use this to compute beyond the Turing (machine) Barrier?
- Do incomputable objects occur in the real world?

Now we witnessed ...
a certain extraordinarily
complicated looking set, namely the
Mandelbrot set.

Although the rules which provide its definition
are surprisingly simple, the set itself exhibits
an endless variety of highly elaborate
structures.



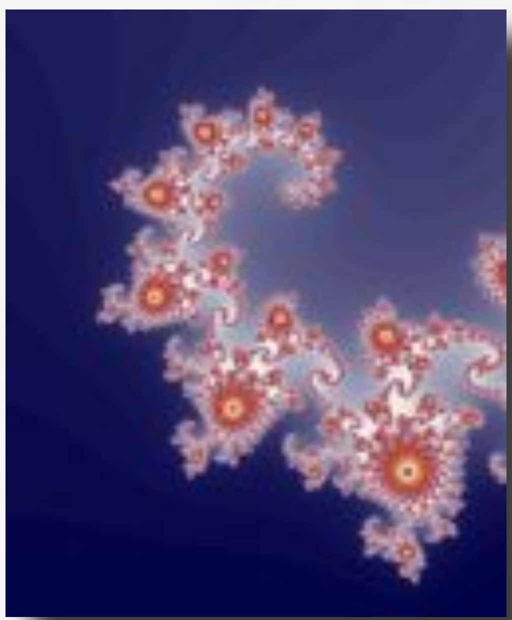


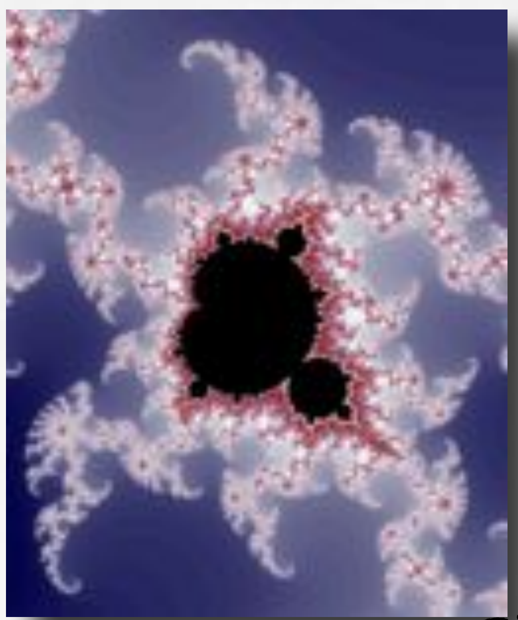
Roger Penrose

in "The Emperor's New mind", Oxford Univ. Press, 1994

OPEN PROBLEM:

Is the Mandelbrot set computable??





Real world randomness

- Quantum randomness is a familiar experimental and theoretical phenomenon

It passes all reasonable statistical properties of randomness

- Cris Calude/Karl Svozil: It is Turing incomputable

Open question: How random is quantum randomness ?

Mathematical reasoning may be regarded ... as the exercise of a combination of ... *intuition* and *ingenuity*. ... In pre-Gödel times it was thought by some that all the intuitive judgements of mathematics could be replaced by a finite number of ... rules. The necessity for intuition would then be entirely eliminated.

In our discussions, however, we have gone to the opposite extreme and eliminated not intuition but ingenuity, and this in spite of the fact that our aim has been in much the same direction.

Alan Turing [1939],

Systems of logic based on ordinals, Proc. London Math. Soc. (2) 45, pp.161-228.

Reprinted in *A. M. Turing, Collected Works: Mathematical Logic*, pp. 81-148.

An explanation of why written proofs do not tell us how the proof was discovered . . .

And mathematical creativity observed

"At first Poincaré attacked [a problem] vainly for a fortnight, attempting to prove there could not be any such function ... [quoting Poincaré]:

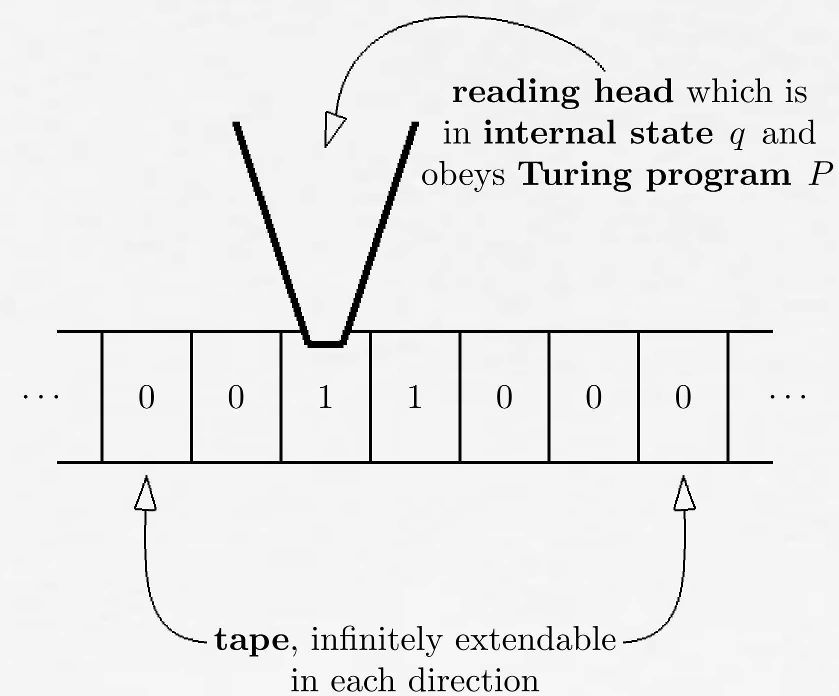
'Having reached Coutances, we entered an omnibus to go some place or other. At the moment when I put my foot on the step, the idea came to me, without anything in my former thoughts seeming to have paved the way for it ... I did not verify the idea ... I went on with a conversation already commenced, but I felt a perfect certainty.

On my return to Caen, for conscience sake, I verified the result at my leisure.' "

from Jacques Hadamard [1945], "The Psychology of Invention in the Mathematical Field", Princeton Univ. Press

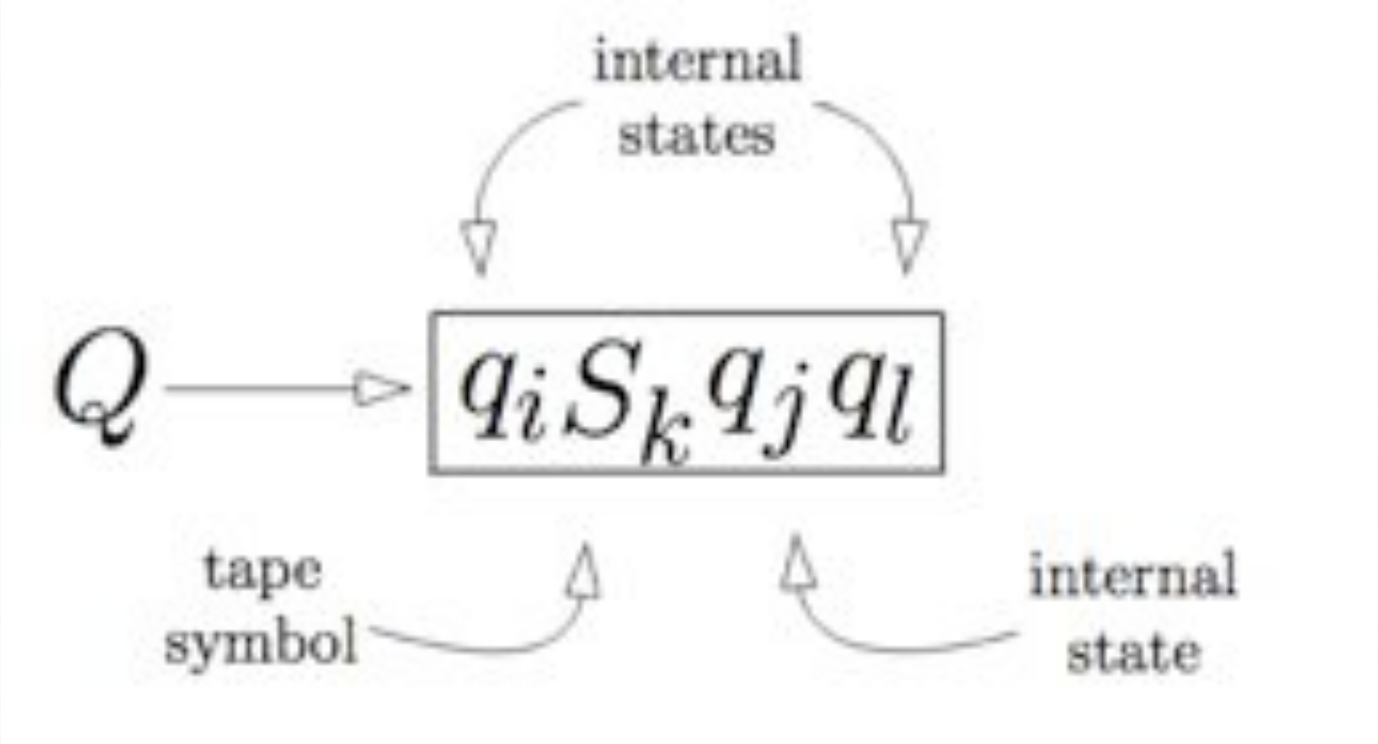
7. Computability and 'Relativity'

- Turing, 1939 - Oracle Turing Machines ...
- Provide a model of how we compute using data given to us from unknown sources
- A model within which Newtonian computability etc comfortably fit ...



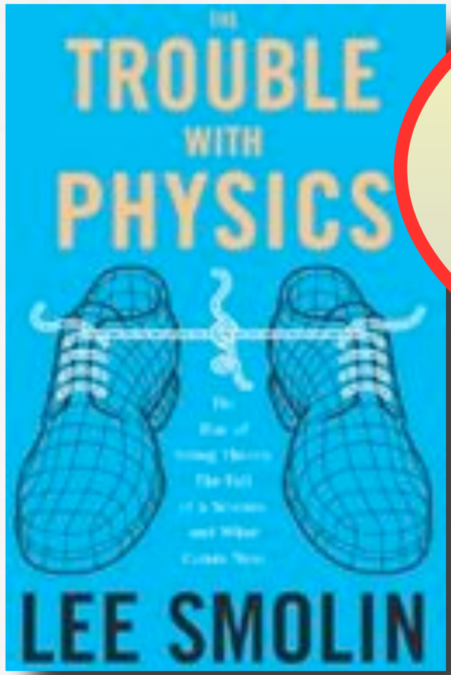
7. Computability and 'Relativity'

- Turing's idea - Allow programs with query quadruples, which can ask for finite information from the 'real world', given by an 'oracle' -



... Causality and Computation

- In modelling the physical universe -



... causality itself is fundamental

Lee Smolin, *'The Trouble With Physics'*, p.241

… fundamental to foundational …

From A. Einstein: "Autobiographical Notes", in "Albert Einstein: Philosopher-Scientist" (P. Schilpp, ed.), Open Court Publishing, 1969, p.63

… I would like to state a theorem which at present can not be based upon anything more than upon a faith in the simplicity, i.e. intelligibility, of nature … nature is so constituted that it is possible logically to lay down such strongly determined laws that within these laws only rationally completely determined constants occur (not constants, therefore, whose numerical value could be changed without destroying the theory) …

problems in physics …

Causality Beyond Computation?

[According to Strong Determinism] … all the complication, variety and apparent randomness that we see all about us, as well as the precise physical laws, are all exact and unambiguous consequences of one single coherent mathematical structure.

Roger Penrose: *Quantum physics and conscious thought*, in *Quantum Implications: Essays in honour of David Bohm* (B.J. Hiley and F.D. Peat, eds.), pp.106-107

Causality Beyond Computation?

If the creation of the universe can be described as a quantum process, we would be left with one deep mystery of existence: What is it that determined the laws of physics?

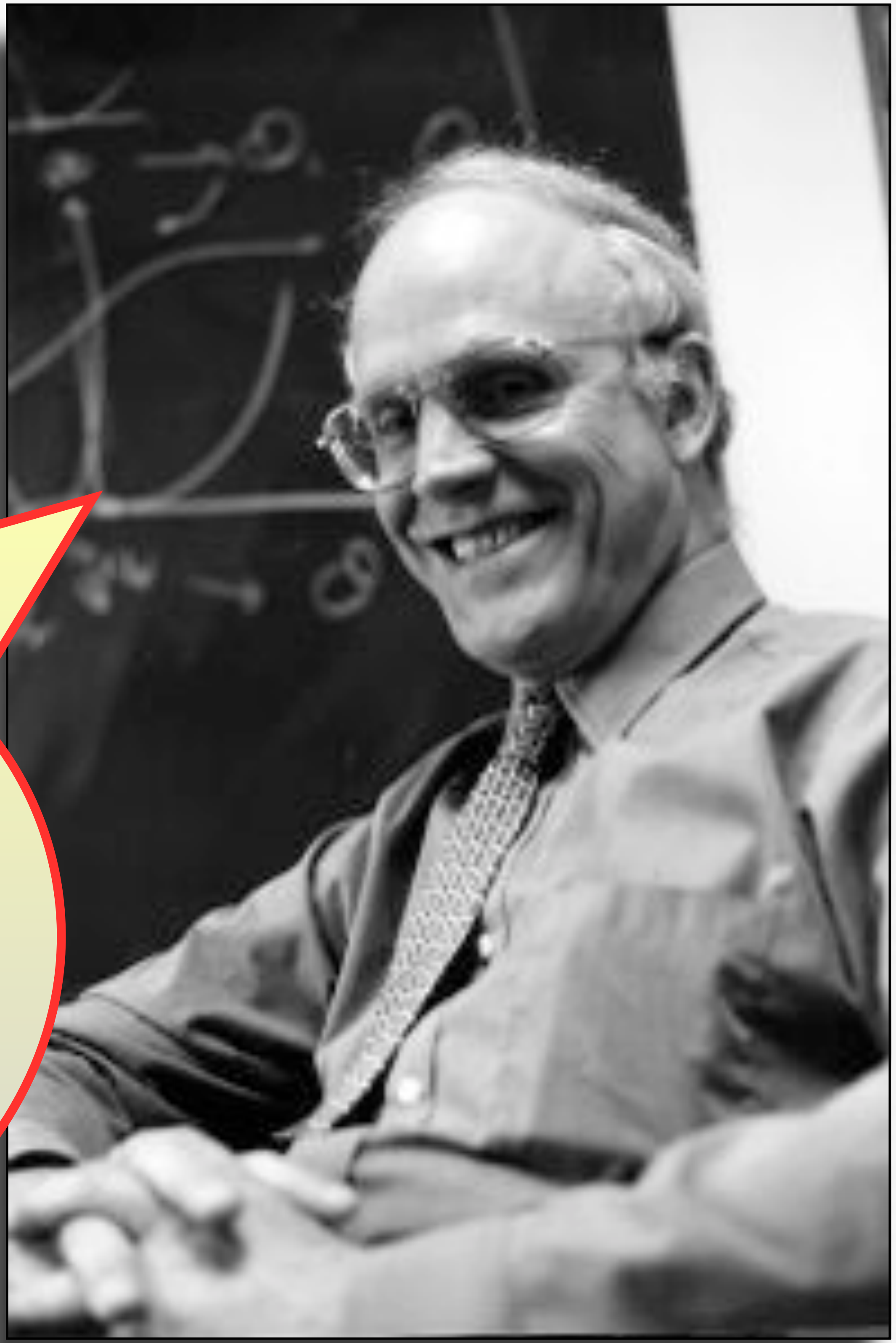
A.H. Guth, *The Inflationary Universe - The Quest for a New Theory of Cosmic Origins*, Addison-Wesley, 1997

One way of thinking about what is unsatisfactory about the standard model is that it leaves seventeen non-trivial numbers still to be explained, ...

Peter Woit: *Not Even Wrong - The Failure of String Theory and the Continuing Challenge to Unify the Laws of Physics*, Jonathan Cape, 2006

David Gross, quoted in New Scientist, Dec. 10 2005, "Nobel Laureate Admits String Theory Is In Trouble":

The state of physics today is like it was when we were mystified by radioactivity ... They were missing something absolutely fundamental. We are missing perhaps something as profound as they were back then.



Causality Beyond Computation?

- *Early champions of the role of causality* - Roger Penrose, Rafael Sorkin, Fay Dowker, and Fotini Markopoulou

> **It is not only the case that the spacetime geometry determines what the causal relations are. This can be turned around: Causal relations can determine the spacetime geometry ...**
> **It's easy to talk about space or spacetime emerging from something more fundamental, but those who have tried to develop the idea have found it difficult to realize in practice. ... We now believe they failed because they ignored the role that causality plays in spacetime. These days, many of us working on quantum gravity believe that causality itself is fundamental - and is thus meaningful even at a level where the notion of space has disappeared.**

Lee Smolin, The Trouble With Physics, p.241

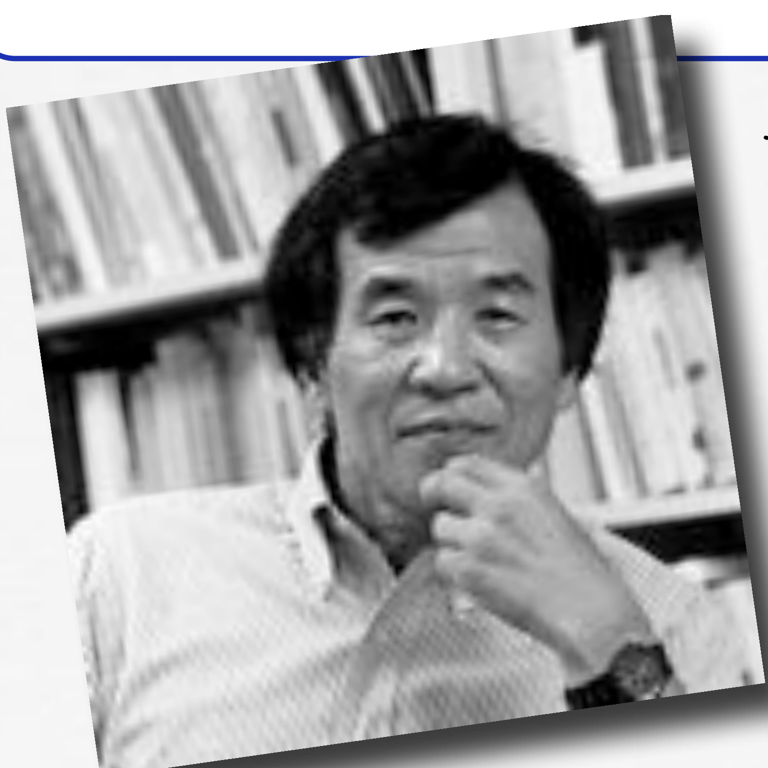
Enter Mathematics of Relative Computability

- 1939 - Turing's oracle Turing machines
- Provide a model of computable content of structures, based on p.c. functionals over the reals
- 1944 - Post defines the degrees of unsolvability as a classification of reals in terms of their relative computability
- Giving a landscape with a rich structure

Phyllis, Emil and Gertrude Post

8. The Brain as Mathematics

Supervenience 'represents the idea that mentality is at bottom physically based, and that there is no free-floating mentality unanchored in the physical nature of objects and events in which it is manifested'



from Jaegwon Kim: "Mind in a Physical World", MIT Press, 1998, pp.14-15

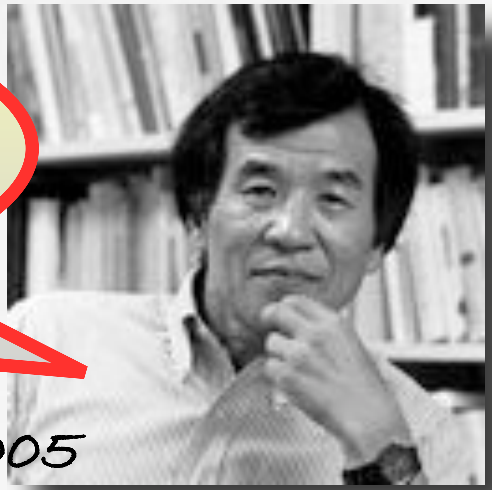
"A set of properties A supervenes upon another set B just in case no two things can differ with respect to A-properties without also differing with respect to their B-properties."

Stanford Encyclopedia of Philosophy

8. The Brain as Mathematics

- How can mentality have a causal role in a world that is fundamentally physical?
- And what about 'overdetermination' – the problem of phenomena having both mental and physical causes?

... the problem of mental causation is solvable only if mentality is physically reducible; however, phenomenal consciousness resists physical reduction, putting its causal efficacy in peril.



–Jaegwon Kim: Physicalism, or Something Near Enough, Princeton, 2005

Connectionist Models to the Rescue?

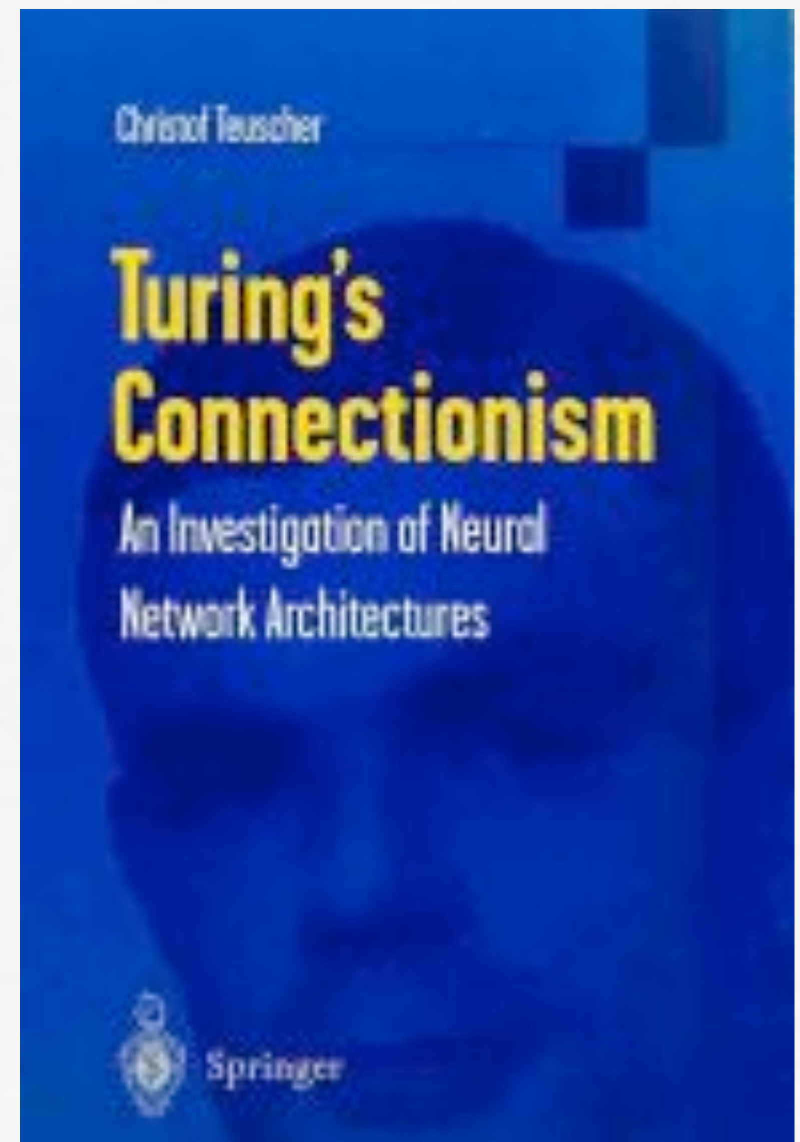
There is a reasonable chance that connectionist models will lead to the development of new somewhat-general-purpose self-programming, massively parallel analog computers, and a new theory of analog parallel computation: they may possibly even challenge the strong construal of Church's Thesis as the claim that the class of well-defined computations is exhausted by those of Turing machines.

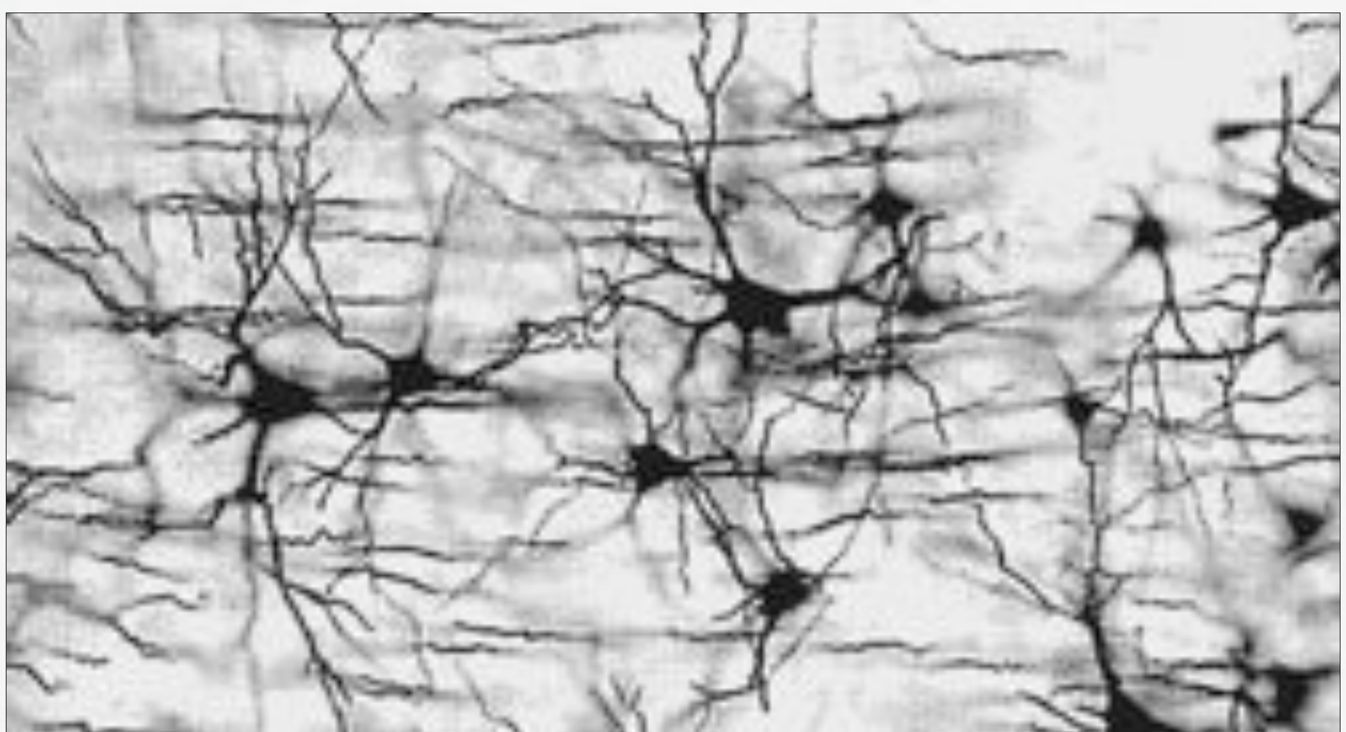
Paul Smolensky [1988] (recipient 2005 David E. Rumelhart Prize),

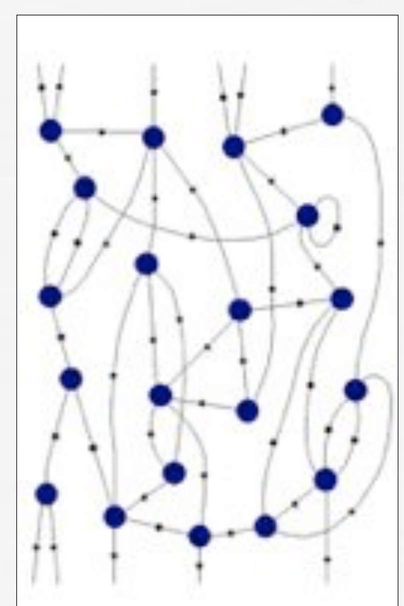
On the proper treatment of connectionism, in Behavioral and Brain Sciences, 11, pp. 1-74

Connectionist Models to the Rescue?

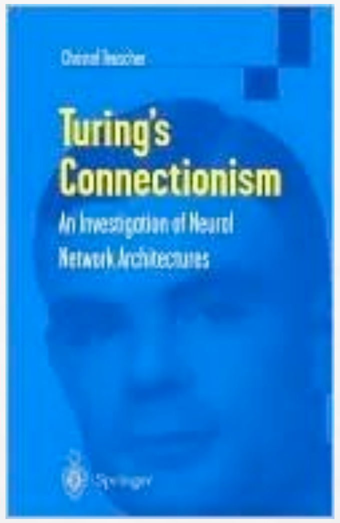
- Turing, 1948: ‘Unorganised machines’= neural nets
- Beginnings: Warren McCulloch and Walter Pitts,1943







Connectionist Models to the Rescue?



- These have come a long way since Turing's [1948] discussion of 'unorganised machines', and McCulloch and Pitts [1943] early paper on neural nets

- But for Steven Pinker "... neural networks alone cannot do the job".

And focussing on our elusive higher functionality, he points to a "kind of mental fecundity called recursion" . . .

Connectionist Models to the Rescue?

We humans can
take an entire proposition and give it a role in some larger proposition. Then we can take the larger proposition and embed it in a still-larger one. Not only did the baby eat the slug, but the father saw the baby eat the slug, and I wonder whether the father saw the baby eat the slug, the father knows that I wonder whether he saw the baby eat the slug, and I can guess that the father knows that I wonder whether he saw the baby eat the slug, and so on.

Steven Pinker,

How the Mind Works, W. W. Norton, New York, 1997

Connectionist Models to the Rescue?

"As the brain forms images of an object - such as a face, a melody, a toothache, the memory of an event - and as the images of the object *affect* the state of the organism, yet another level of brain structure creates a swift nonverbal account of the events that are taking place in the varied brain regions activated as a consequence of the object-organism interaction. The mapping of the object-related consequences occurs in first-order neural maps representing the proto-self and object; the account of the *causal relationship* between object and organism can only be captured in second-order neural maps. ... one might say that the swift, second-order nonverbal account narrates a story: ***that of the organism caught in the act of representing its own changing state as it goes about representing something else.***"

- Antonio Damasio [1999], The Feeling Of What Happens, p.170

9. Intelligent Machines?

- Solomon Feferman, 1988: "Turing in the Land of O(z)", in "The Universal Turing Machine: A Half-Century Survey" (Herken R., ed.), Oxford University Press, pp.131–2:

"Turing, as is well known, had a mechanistic conception of mind, and that conviction led him to have faith in the possibility of machines exhibiting intelligent behavior."

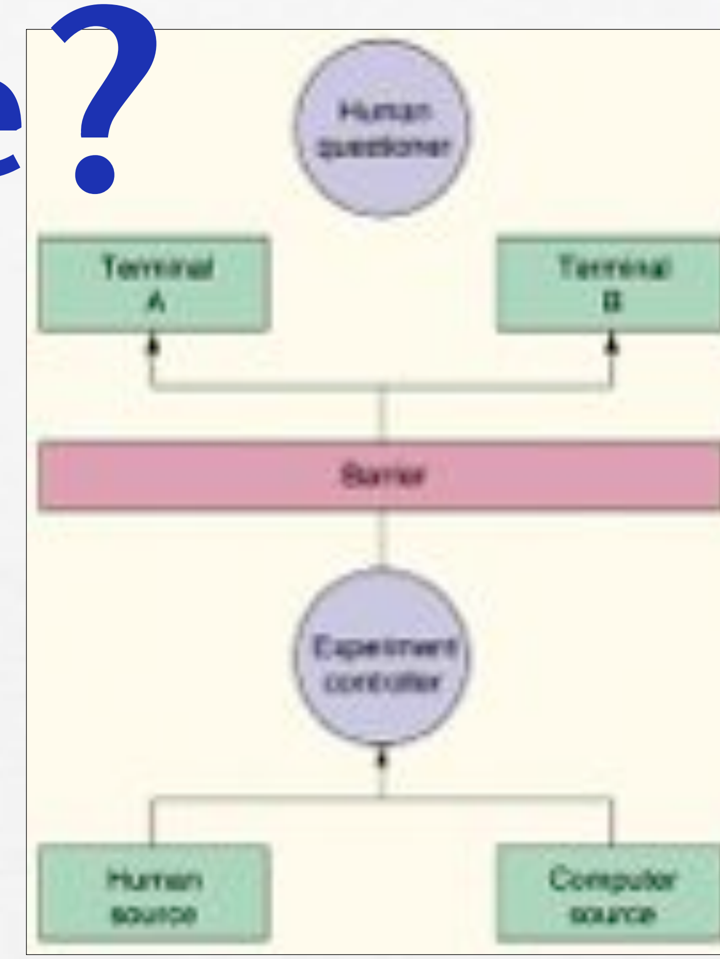
- But what is intelligence? "… if a machine is expected to be infallible, it cannot also be intelligent. There are several theorems which say almost exactly that."

– A.M. Turing, talk to the London Mathematical Society, February 20, 1947, quoted in Andrew Hodges, p.361

- "The results which have been described in this article are mainly of a negative character, setting certain bounds to what we can hope to achieve purely by reasoning. These, and some other results of mathematical logic may be regarded as going some way towards a demonstration, within mathematics itself, of the inadequacy of 'reason' unsupported by common sense."

– final paragraph of Alan Turing, Solvable and Unsolvable Problems, Penguin Science News 31, 1954, p.23

A Test for Machine Intelligence?



- How do we judge whether a machine can "think" at approximately human-level?

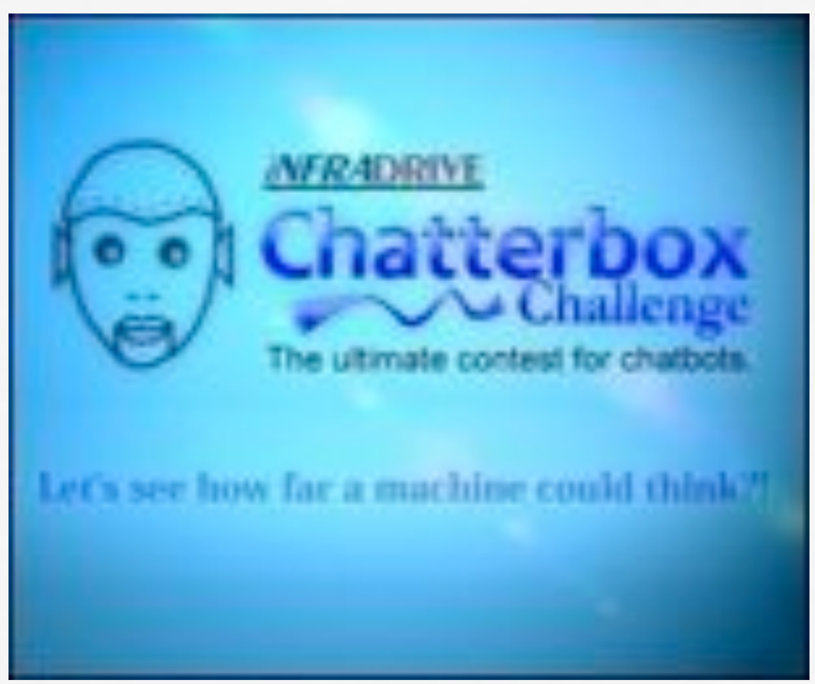
- The Turing Test (1950): A human examiner 'converses' with another human, and with the machine (an advanced computer), without knowing which is which.

- If the examiner fails to correctly identify which is the machine, then the machine passes the test.

Practical 'Imitation Games' ...

The Loebner Prize - $100,000 at stake since 1990 ...

- Chatterbox Challenge, from 2001

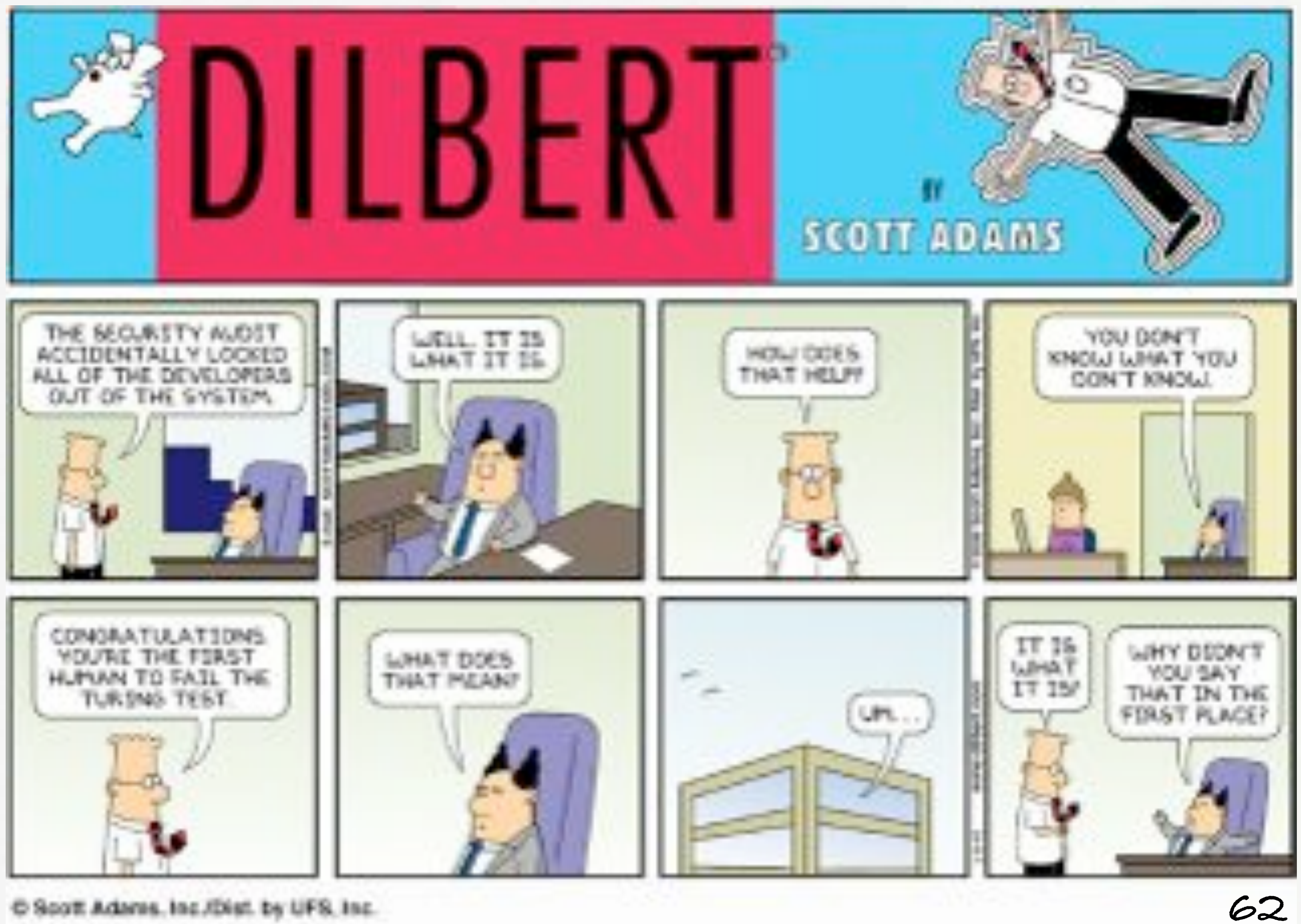


'Elbot' by Fred Roberts - Loebner 1st Place, 2008, talks online: http://www.elbot.com/

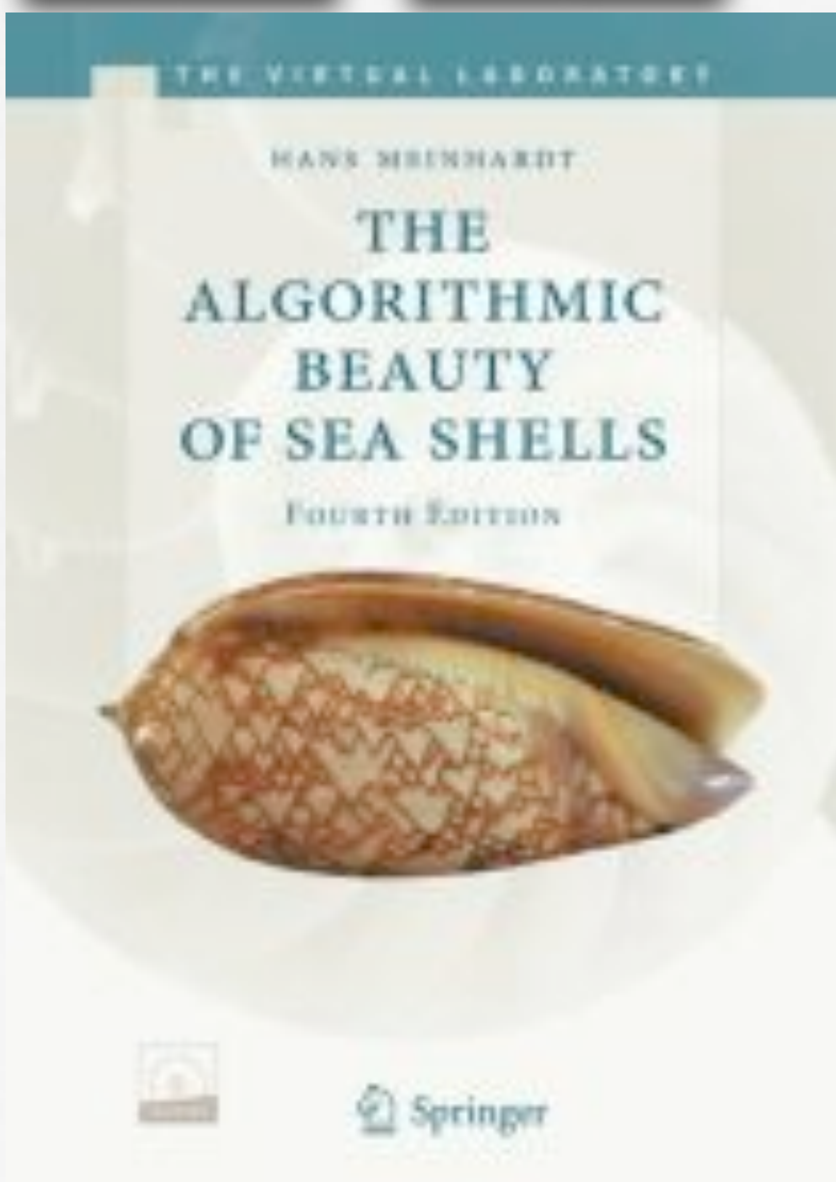
- Rodney Brooks in Nature, 2001:

... neither AI nor Alife has produced artifacts that could be confused with a living organism for more than an instant.

Turing Test in Popular Culture ...

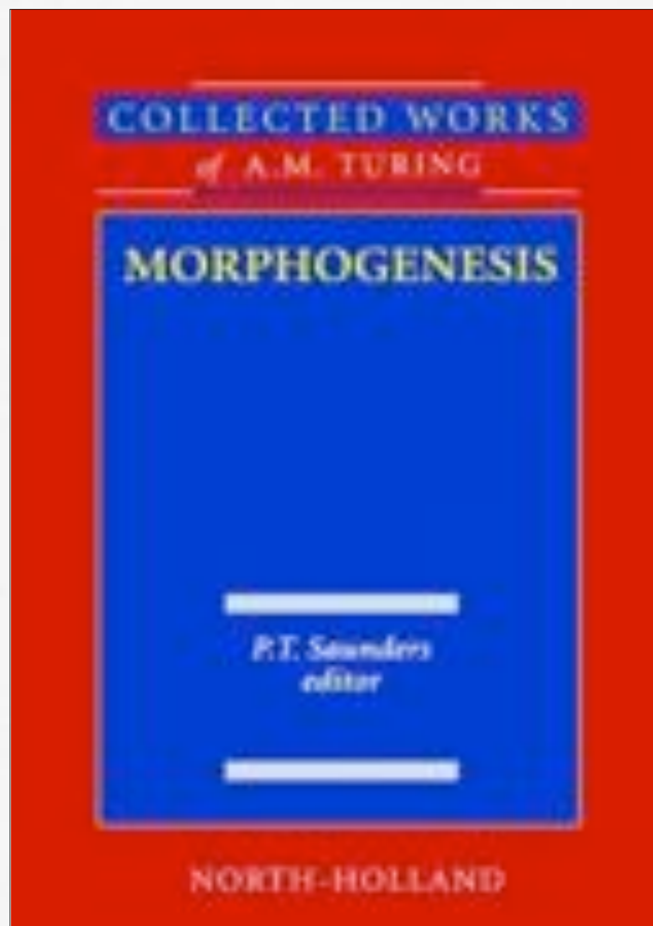


10. How Nature Computes



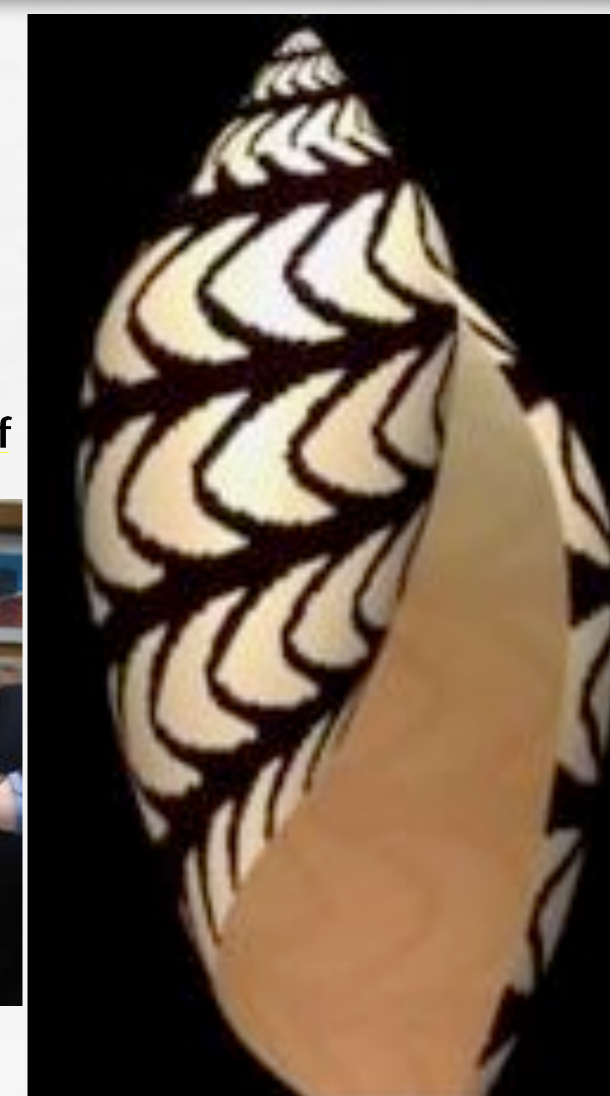
10. How Nature Computes

1950s - Alan Turing proposes a simple reaction-diffusion system describing chemical reactions and diffusion to account for morphogenesis, i.e., the development of form and shape in biological systems.



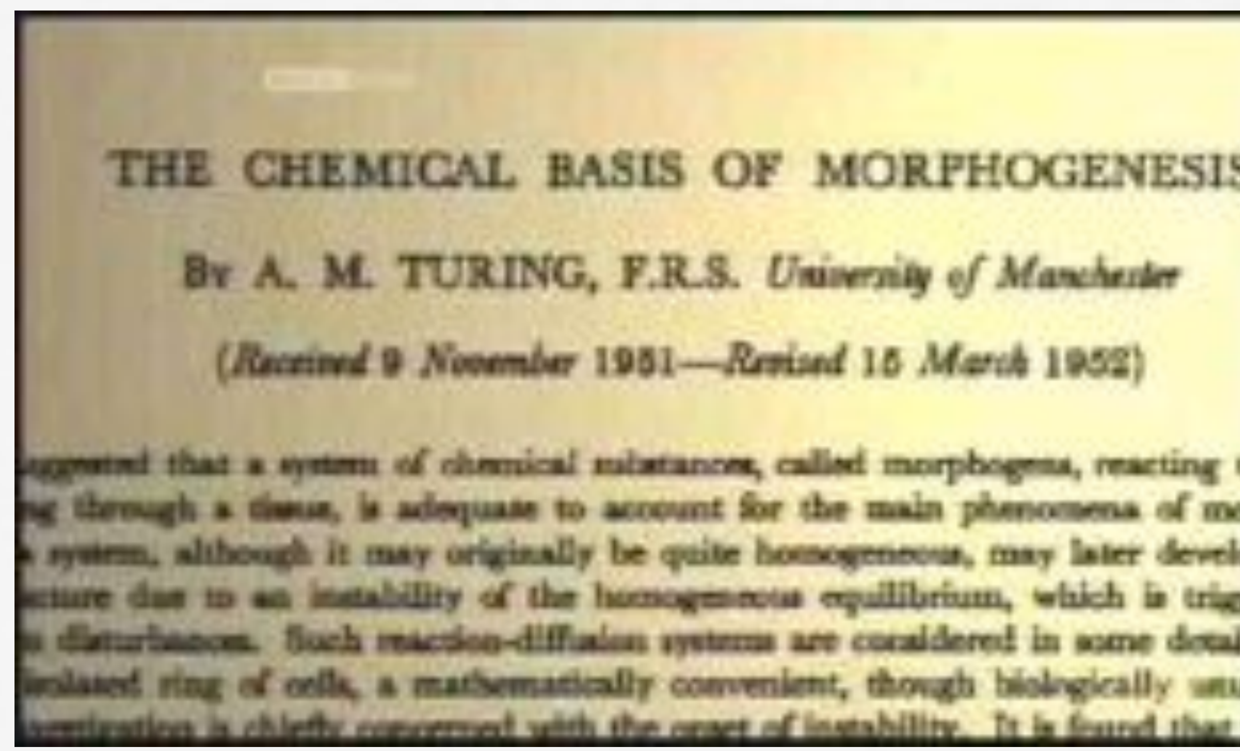
From website of the Biological Modeling and Visualization research group, Department of Computer Science at the University of Calgary:

Przemyslaw Prusinkiewicz



See http://www.swintons.net/jonathan/turing.htm

10. How Nature Computes



THE CHEMICAL BASIS OF MORPHOGENESIS

By A. M. TURING, F.R.S. *University of Manchester*

(*Received* 9 *November* 1951—*Revised* 15 *March* 1952)

Emergence and Self-Organisation

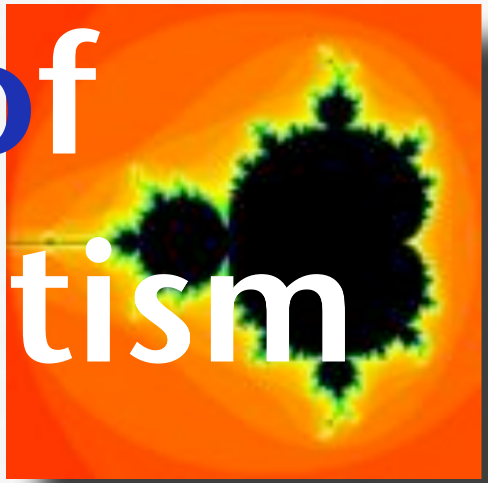
Hermann Haken

- SYNERGETICS - the study of the origins and evolution of macroscopic patterns and spacio-temporal structures in interactive systems
- Emphasis on mapping out self-organisational processes in science and the humanities - e.g. autopoiesis
- Mathematical modelling of nonlinear and irreversable processes, dissipative structures ...

Ilya Prigogine

See: Michael Bushev, *Synergetics - Chaos, Order, Self-Organization,* World Scientific, 1994

The Rise & Fall of British Emergentism



" ... the characteristic behaviour of the whole ... <u>could</u> not, even in theory, be deduced from the most complete knowledge of the behaviour of its components ... This ... is what I understand by the 'Theory of Emergence'. I cannot give a conclusive example of it, since it is a matter of controversy whether it actually applies to anything ... I will merely remark that, so far as I know at present, the characteristic behaviour of Common Salt cannot be deduced from the most complete knowledge of the properties of Sodium in isolation; or of Chlorine in isolation; or of other compounds of Sodium, ..."

- C.D. Broad, The Mind and Its Place In Nature, Kegan-Paul, London, 1925, p.59

C. D. Broad 1887-1971

Descriptions and Emergent Structure

- Notice - It is often possible to get descriptions of emergent properties in terms of the elementary actions
- E.g., this is what Turing did for the role of Fibonacci numbers in relation to the sunflower etc.
- In mathematics, it is well-known that complicated descriptions may entail incomputable phenomena
- A potential source of surprise in emergence ...

Descriptions and Emergent Structure

- Intuition - entities exist because of, and according to, mathematical laws. In the words of Leibniz [1714] -
- 'The Monadology', section 32:
  " ... there can be found no fact that is true or existent, or any true proposition, without there being a sufficient reason for its being so and not otherwise, although we cannot know these reasons in most cases."

Definability in the Real World

Hans Reichenbach (1891-1953)

- That is - natural phenomena not only generate descriptions, but arise and derive form from them . . .

... so - connecting with a useful abstraction - that of mathematical definability - or, more generally, invariance (under the automorphisms of the appropriate structure) ...

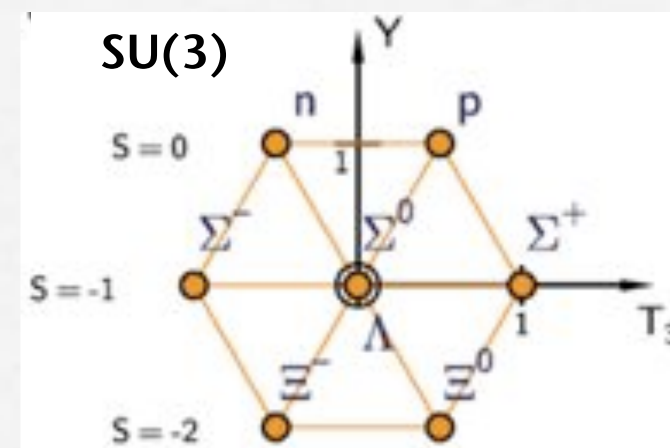
Alfred Tarski

- ... giving precision to our experience of emergence as a potentially non-algorithmic determinant of events

Definability and Symmetries

Murray Gell-Mann

- Symmetries play a huge role in science ..
- ... expressing appropriate automorphisms
- ... or particular lapses in definability



... so giving a clear route: from fundamental mathematical structures, and their automorphisms and breakdowns in definability - to far-reaching macro-symmetries in nature

the expected robustness ...

I believe the following aspects of evolution to be true, without knowing how to turn them into (respectable) research topics.

Important steps in evolution are robust. Multicellularity evolved at least ten times. There are several independent origins of eusociality. There were a number of lineages leading from primates to humans. If our ancestors had not evolved language, somebody else would have.

Martin Nowak,

Director, Program for Evolutionary Dynamics, Harvard University,

in John Brockman (ed.): "What We Believe But Cannot Prove"

Coutances revisited ...

"At first Poincaré attacked [a problem] vainly for a fortnight, attempting to prove there could not be any such function ... [quoting Poincaré]:

'Having reached Coutances, we entered an omnibus to go some place or other. At the moment when I put my foot on the step, the idea came to me, without anything in my former thoughts seeming to have paved the way for it ... I did not verify the idea ... I went on with a conversation already commenced, but I felt a perfect certainty.
On my return to Caen, for conscience sake, I verified the result at my leisure.' "

from Jacques Hadamard [1945], "The Psychology of Invention in the Mathematical Field", Princeton Univ. Press

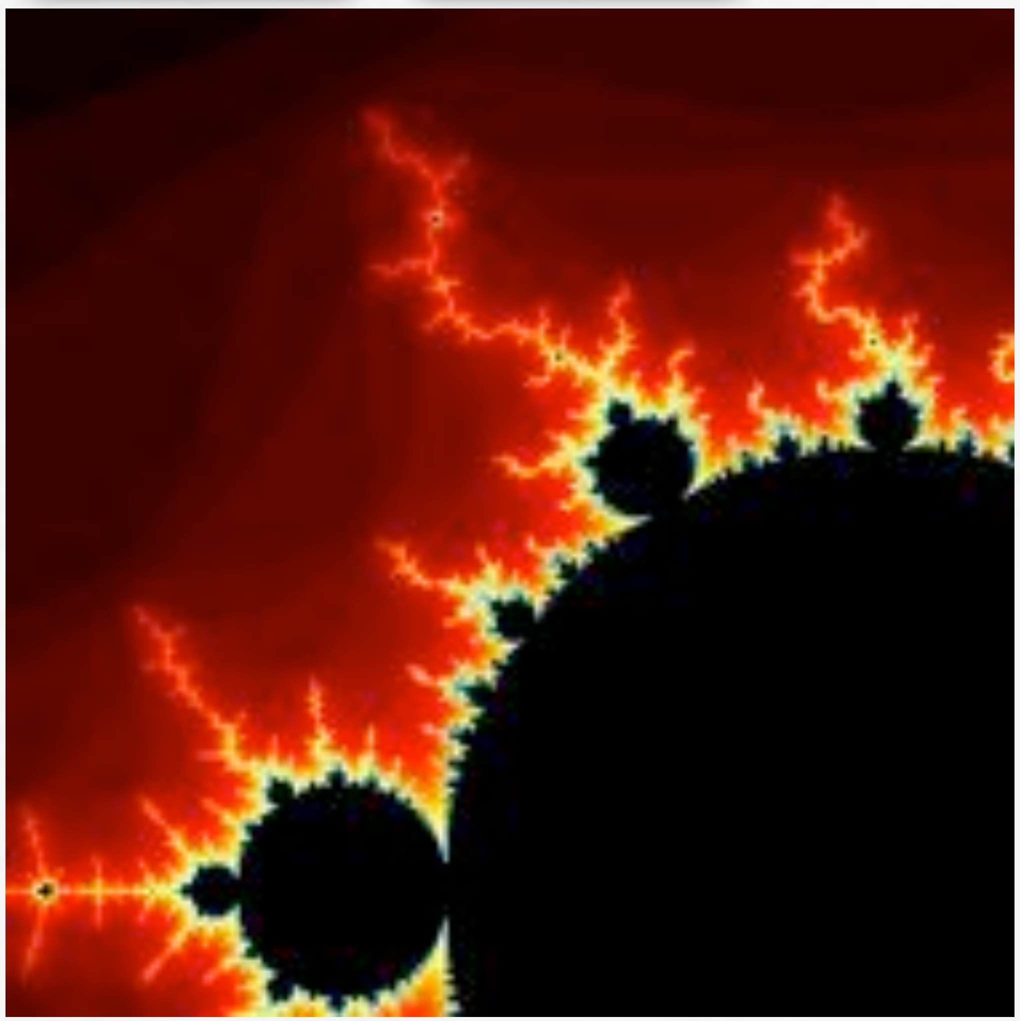
Basic causality, Turing landscape and emergence ...

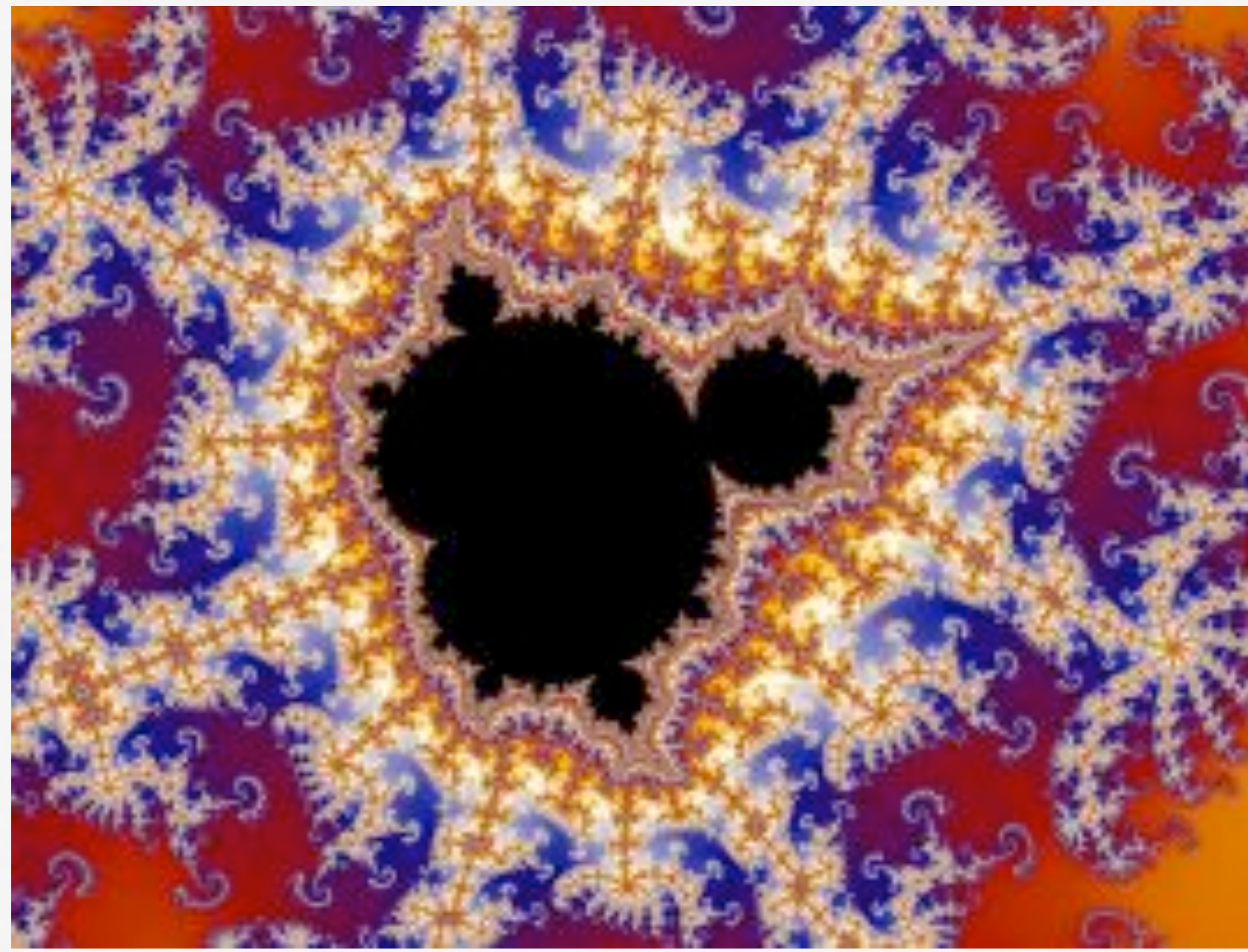
- Can describe global relations in terms of local structure ...
- ... so capturing the emergence of large-scale formations

➡ Mathematically - formalise as definability over structure based on Turing functionals

➡ More generally - as Invariance under automorphisms

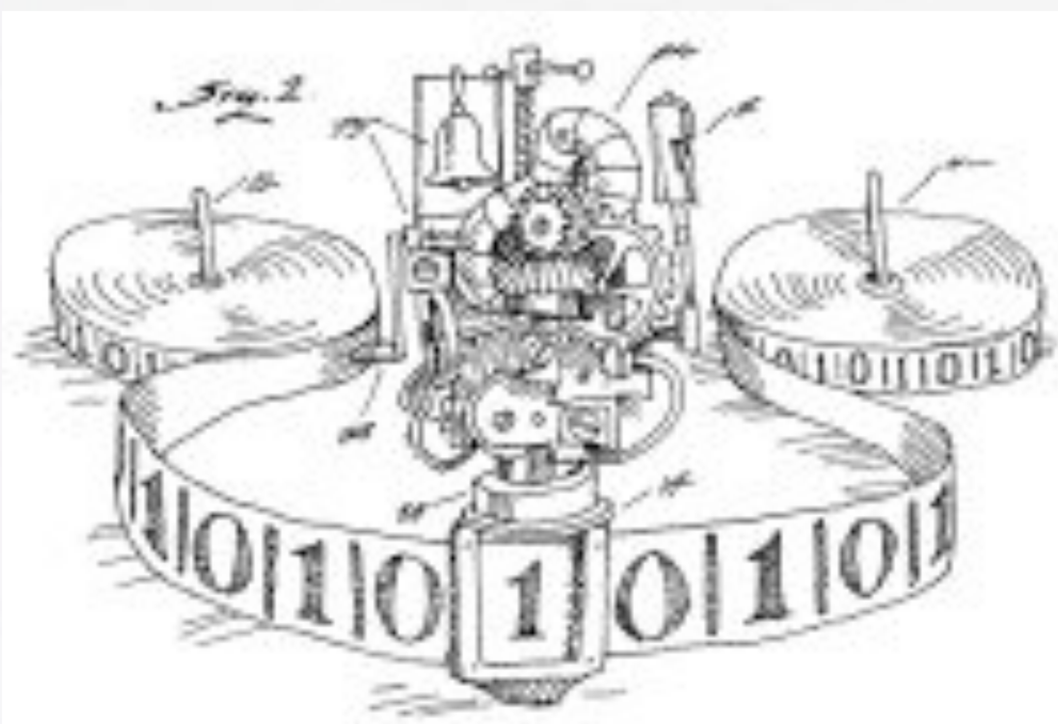
Morphogenesis - in same world as ...

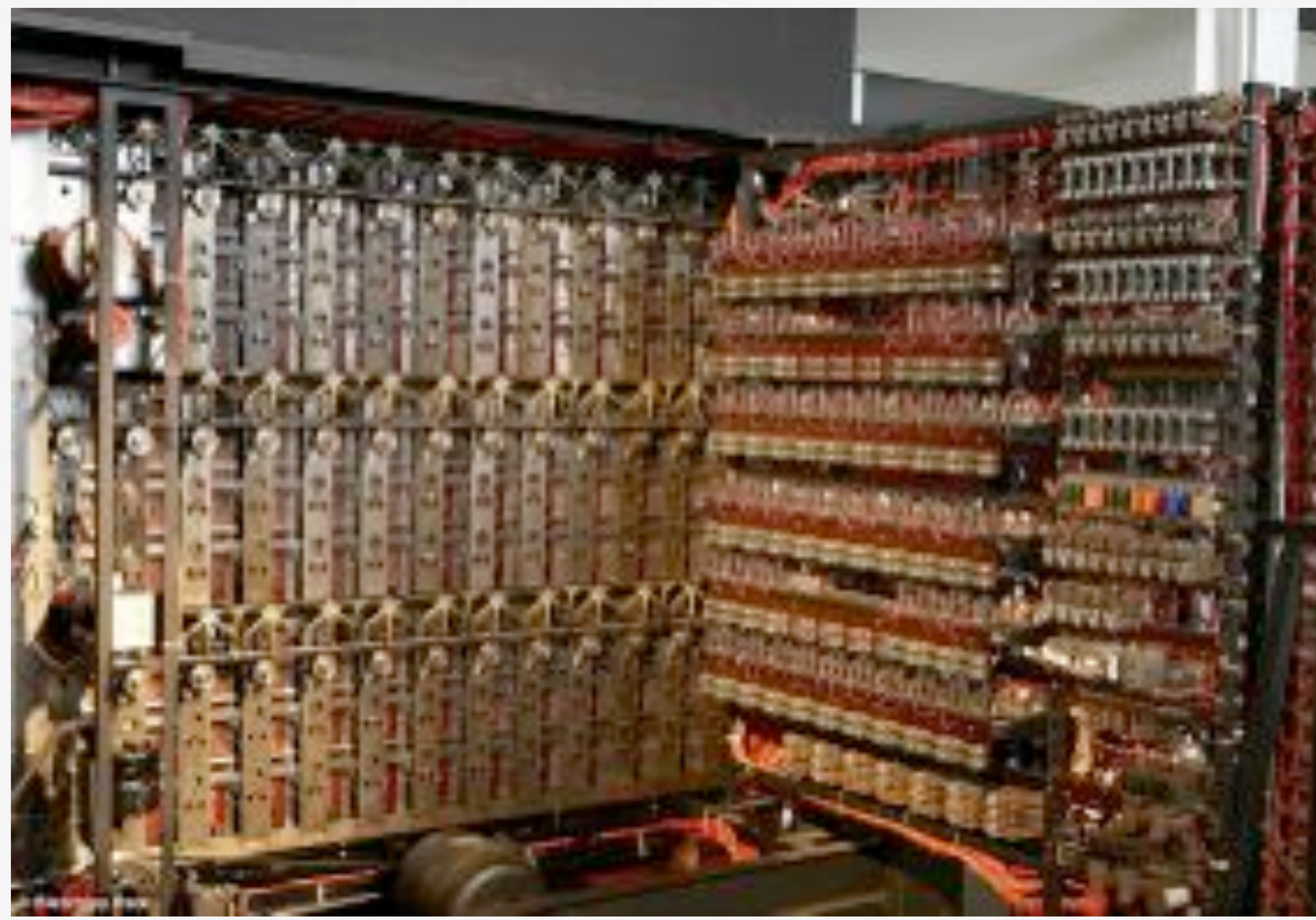




... the Mandelbrot set

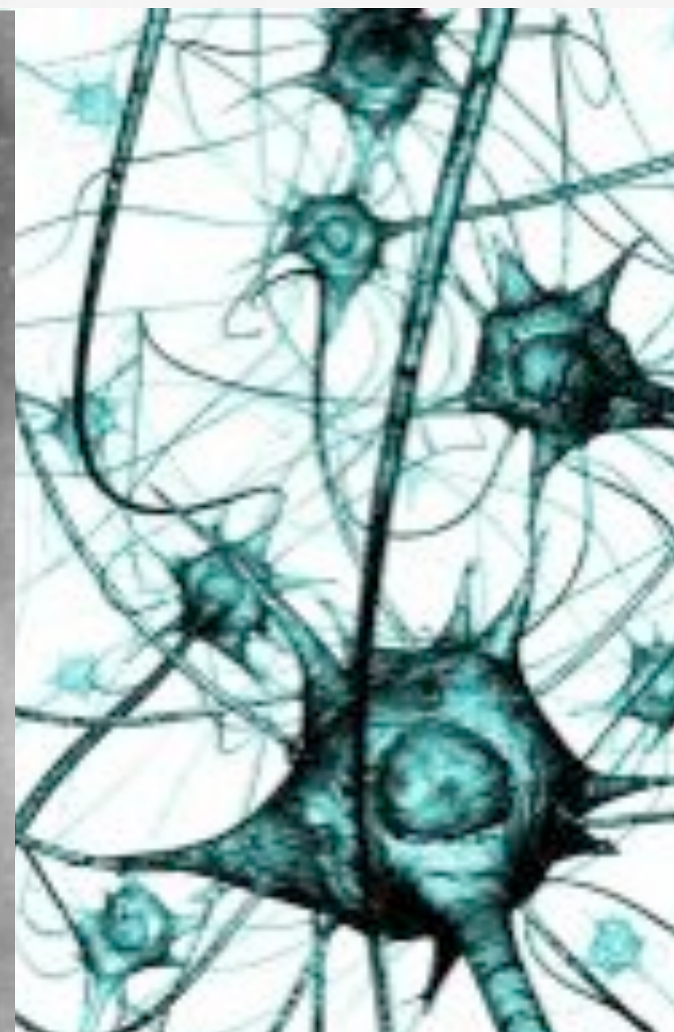
Morphogenesis - in same world as ...





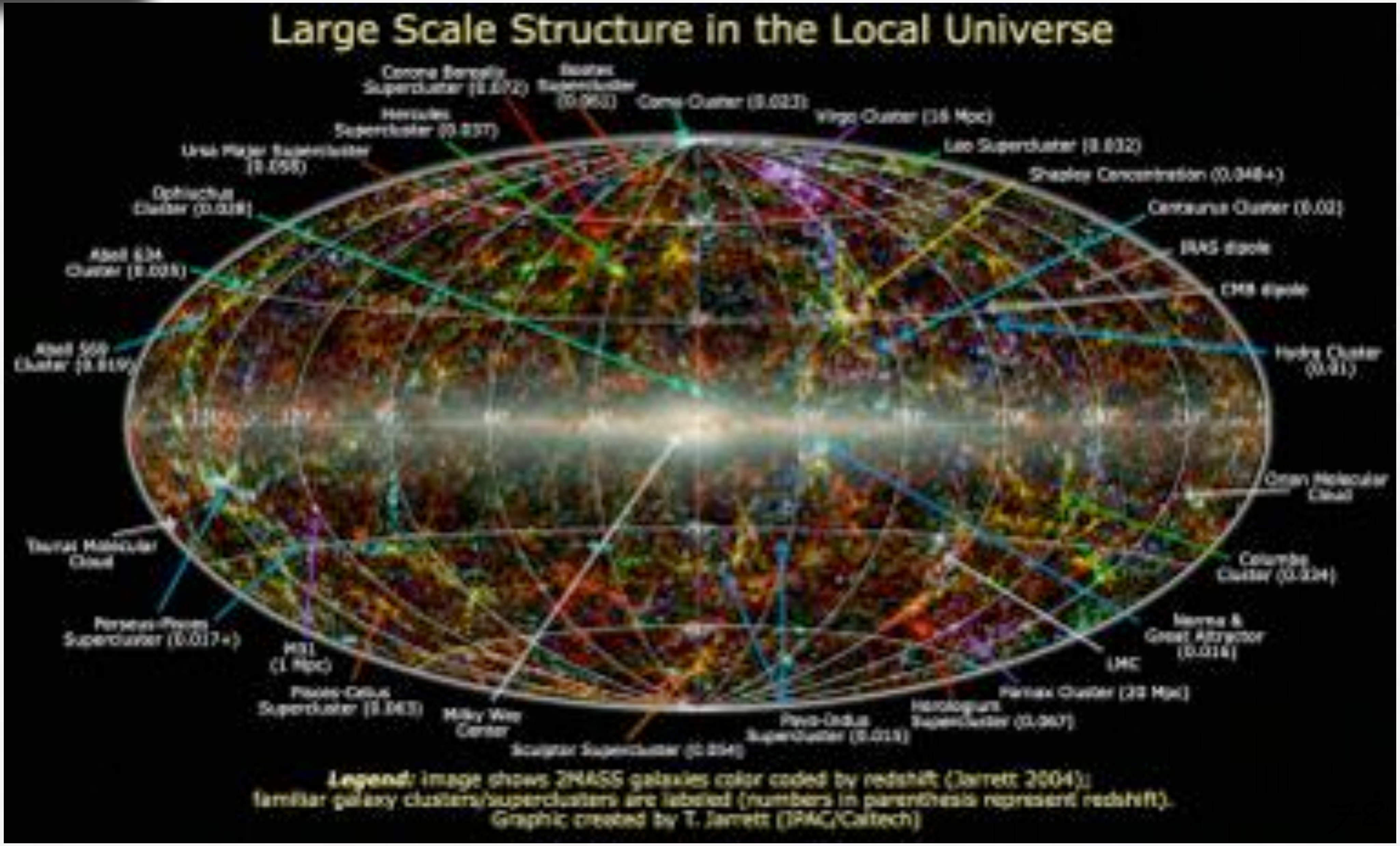
... the Halting Problem

Morphogenesis - in same world as ...


... Mental Supervenience

Morphogenesis - in same world as …



Simple Rules ...

... Complex Outcomes ...

... Emergent Forms ...

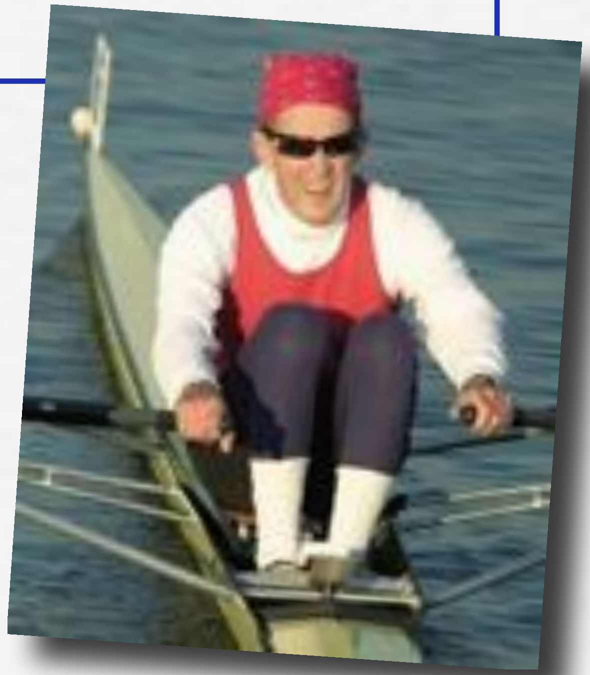
... at the Edge of Computability

And computation via abundant embodied information ...

Cat's Eye Nebula

Hartley Rogers' programme ...

Fundamental problem: Characterise the Turing invariant relations



- Intuition: These are key to pinning down how basic laws and entities emerge as mathematical constraints on causal structure

- Notice: The richness of Turing structure discovered so far becomes the raw material for a multitude of non-trivially definable relations

Bi-interpretability

Bi-interpretability Conjecture(Harrington): The Turing definable relations are exactly those with information content describable in second-order arithmetic

- Where: Conjecture rules out there being non-trivial Turing automorphisms …

- While: Partial results underpin observed certainties …

Examples of Failure of Definability?

Processes for change of wave equation describing quantum state of a physical system:

- Deterministic continuous evolution via Schrödinger's equation - involves superpositions of basis states
- Probabilistic non-local discontinuous change due to measurement - observe a jump to a single basis state

Putative mathematical explanation in terms of assertion of previously imperfect definability ... providing ...



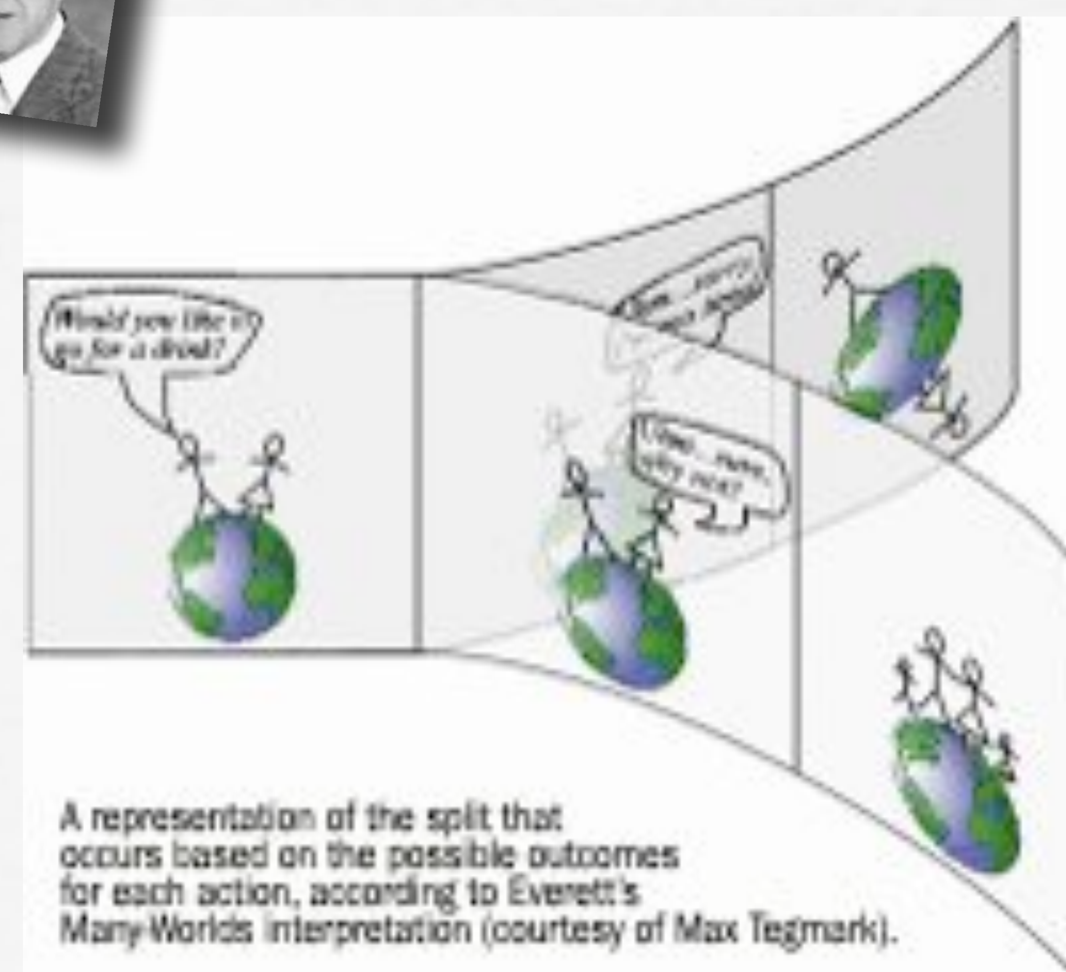
... an alternative to Many Worlds

Bryce DeWitt

John Wheeler

Hugh Everett III

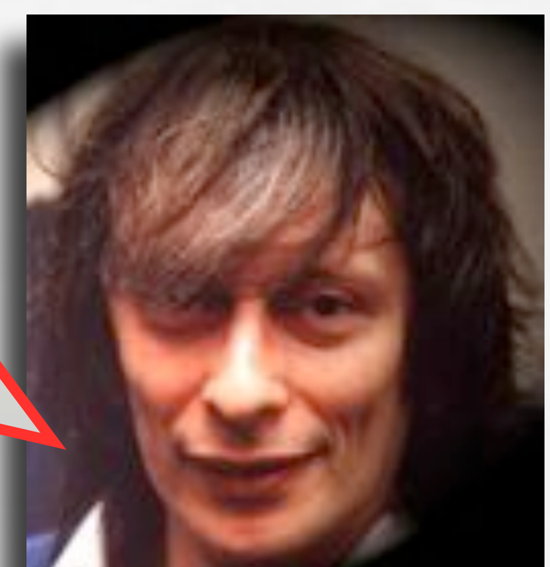
(Nov.11, 1930- July 19, 1982)



A representation of the split that occurs based on the possible outcomes for each action, according to Everett's Many-Worlds interpretation (courtesy of Max Tegmark).

and the Multiverse +Anthropic Principle

... understanding the multiverse is a precondition for understanding reality as best we can. Nor is this said in a spirit of grim determination to seek the truth no matter how unpalatable it may be ... It is, on the contrary, because the resulting world-view is so much more integrated, and makes more sense in so many ways, than any previous world-view, and certainly more than the cynical pragmatism which too often nowadays serves as surrogate for a world-view amongst scientists.

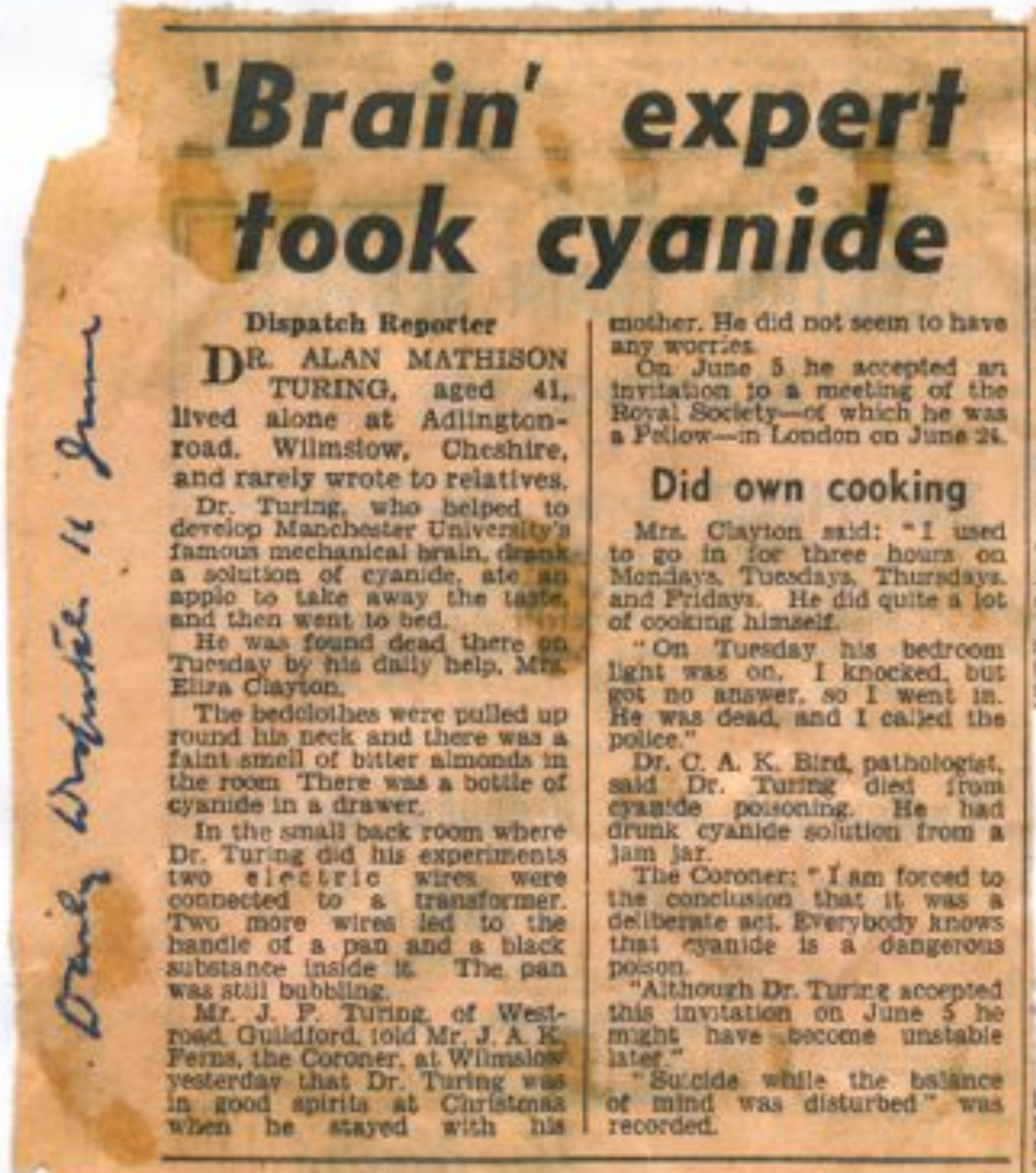


David Deutsch, *The Fabric of Reality*, Allen Lane, 1997, p.48

And: Downward Causation Revisited

The "levels" involved are levels of organisation and integration, and the downward influence means that *the behavior of "lower" levels - that is, of the components of which the "higher-level" structure consists - is different than it would otherwise be, because of the influence of the new property that emerges in consequence of the higher-level organization.*

William Hasker, *'The Emergent Self'*, Cornell University Press, 1999, p.175



'Brain' expert took cyanide

Dispatch Reporter

DR. ALAN MATHISON TURING, aged 41, lived alone at Adlington-road, Wilmslow, Cheshire, and rarely wrote to relatives.

Dr. Turing, who helped to develop Manchester University's famous mechanical brain, drank a solution of cyanide, ate an apple to take away the taste, and then went to bed.

He was found dead there on Tuesday by his daily help, Mrs. Eliza Clayton.

The bedclothes were pulled up round his neck and there was a faint smell of bitter almonds in the room. There was a bottle of cyanide in a drawer.

In the small back room where Dr. Turing did his experiments two electric wires were connected to a transformer. Two more wires led to the handle of a pan and a black substance inside it. The pan was still bubbling.

Mr. J. F. Turing, of West-road, Guildford, told Mr. J. A. K. Ferns, the Coroner, at Wilmslow yesterday that Dr. Turing was in good spirits at Christmas when he stayed with his mother. He did not seem to have any worries.

On June 5 he accepted an invitation to a meeting of the Royal Society—of which he was a Fellow—in London on June 24.

Did own cooking

Mrs. Clayton said: "I used to go in for three hours on Mondays, Tuesdays, Thursdays, and Fridays. He did quite a lot of cooking himself.

"On Tuesday his bedroom light was on. I knocked, but got no answer, so I went in. He was dead, and I called the police."

Dr. C. A. K. Bird, pathologist, said Dr. Turing died from cyanide poisoning. He had drunk cyanide solution from a jam jar.

The Coroner: "I am forced to the conclusion that it was a deliberate act. Everybody knows that cyanide is a dangerous poison.

"Although Dr. Turing accepted this invitation on June 5 he might have become unstable later."

"Suicide while the balance of mind was disturbed" was recorded.

2009 - the Prime Minister's Apology

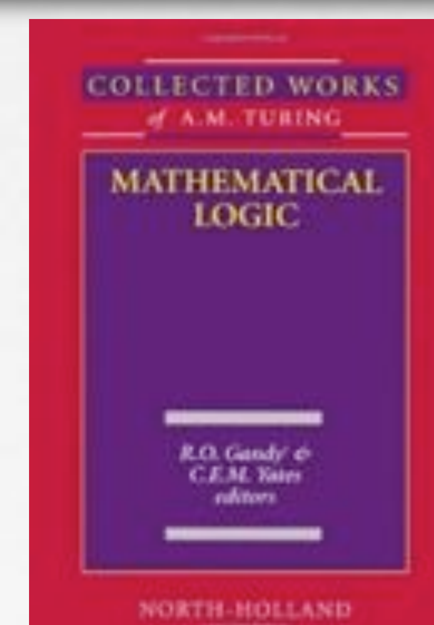
Gordon Brown issued an unequivocal apology last night on behalf of the government to Alan Turing, the second world war codebreaker who took his own life 55 years ago after being sentenced to chemical castration for being gay.

Describing Turing's treatment as "horrifying" and "utterly unfair", Brown said the country owed the brilliant mathematician a huge debt. He was proud, he said, to offer an official apology. "We're sorry, you deserved so much better," Brown writes in a statement posted on the No 10 website.

theguardian

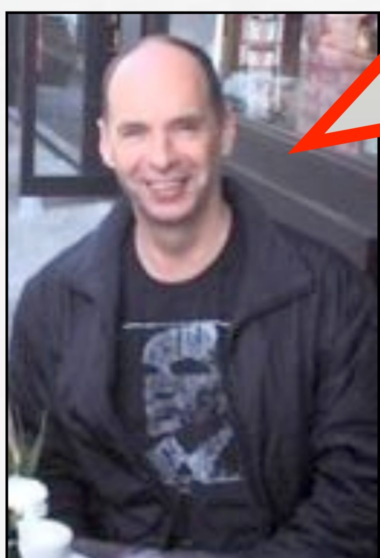
"Some things happen for no reason ..."

- Robin Gandy (1919-95)



It is the greatest honour to unveil the place where Ethel Sara gave birth on 23 June 1912. My sentiments are not the same as those that impelled her to rediscover her dead son and tell her story. But I hope she would recognise an echo of her language when I conclude:

The law killed but the spirit gives life.



Andrew Hodges

www.turing.org.uk

unveiling the Blue Plaque at the Colonnade Hotel, June 23rd, 1998

Thank you!